Modeling the Distribution of Tariff Revenue to Firms within a Partial Equilibrium Framework

Paul Phillips

ECONOMICS WORKING PAPER SERIES

Working Paper 2025-08-B

U.S. INTERNATIONAL TRADE COMMISSION

500 E Street SW

Washington, DC 20436

August 2025

Modeling the Distribution of Tariff Revenue to Firms within a Partial Equilibrium Framework

Paul Phillips

Economics Working Paper 2025-08-B

August 2025

Abstract

In this paper, I expand upon the current partial equilibrium modeling literature by discussing how to incorporate the distribution of tariff revenue to firms into these models. Rather than assume all firms are identical, I model firms as having a continuum of heterogeneous productivities, so that model users can compute equilibrium results on the fraction of firms that enter or leave the market in any given counterfactual situation. I then solve the model under various counterfactual scenarios given theoretical parameter choices, and illustrate the ways in which modeling outcomes are sensitive to parameter values. While this paper's simulations do not use any real-world data or correspond to any real-world countries, its calibration and estimation discussion takes into account current data limitations and practical challenges in solving the model.

Paul Phillips

Research Division, Office of Economics

U.S. International Trade Commission

Paul.Phillips@usitc.gov

Suggested citation:

Phillips, Paul. ”Modeling the Distribution of Tariff Revenue to Firms within a Partial Equilibrium Framework.”

U.S. International Trade Commission. Office of Economics Working Paper 2025-08-B.

1 Introduction

The revenue generated by tariffs may be distributed in several different ways, with potential recipients including the government, consumers, or firms. Examples of policies that distributed tariff revenue to firms include the 2019 Market Facilitation Program, which provided direct payments to farmers impacted by trade disruptions, and the Byrd Amendment, which distributed the revenue generated from anti-dumping or countervailing duties among firms that filed the original anti-dumping or subsidy complaints.[1] Although the distribution of tariff revenue to firms has been implemented as a policy, the international trade literature has given significantly less attention to modeling it. In this paper, I contribute to existing scholarship on partial equilibrium modeling by introducing a method that incorporates tariff transfer payments to firms within a static, single-stage partial equilibrium model.

The model introduced in this paper can represent any number of countries or industries, as well as any particular choice of country or industry. Lump-sum transfer payments in the model either arrive unexpectedly after firms have already made decisions about prices and production, or firms know they are receiving some form of payment and take this expected transfer into account when making production decisions. The modeling framework also accommodates different methods for calculating the payments; for example, firms could receive only the tariff revenue collected on imports in their specific industry, or in a multi-industry setup they could receive some proportion of the tariff revenue collected across all imports in the economy.

Consumers in the model substitute between domestically produced goods and imports across any number of foreign countries, with tariffs applying to foreign imports. I assume that total consumption from all sources remains constant in any potential counterfactual. Monopolistically competitive firms decide what quantities to send to each destination, and must pay a fixed cost in order to send their production along that route. Unlike the majority of partial equilibrium models in the literature, this model follows Melitz (2003) by featuring heterogeneous firms with varying productivities, and each origin-destination pair ji has a cutoff productivity level below which firms in country j cannot export to i. This firm heterogeneity allows model users to analyze how lump-sum transfer payments based on tariff revenue would affect the fraction of firms able to produce, an analysis that would not be possible in a model with perfect competition or monopolistic competition with identical firms. Lump-sum payments effectively lower the fixed costs necessary for operating, reducing the minimum productivity necessary for firms to stay in business.

I discuss how to compute equilibria in counterfactual situations where tariffs increase. Due to the difficulty of calibrating certain model parameters, this computational method solves for the percentage changes between baseline and counterfactual equilibria, rather than the absolute values of counterfactual equilibria.

[1] For more information on the Market Facilitation Program, see the following USDA guide. For more information on the Byrd Amendment, see the following information provided by the USITC.

I then illustrate the model's capabilities by applying this method under different theoretical scenarios. As part of the discussion, I analyze the difference in equilibria between theoretical simulations in which transfer payments arrive unexpectedly and theoretical simulations in which firms factor these payments into their production decisions.

The rest of the paper proceeds as follows. Section 2 introduces a partial equilibrium model with heterogeneous firm productivities, while Section 3 goes through what the lump-sum distribution of tariff revenue looks like within the model. Section 4 explains how to solve the model given hypothetical data and parameters, and Section 5 presents the results of solving the model using hypothetical parameters. Section 6 concludes.

1.1 Literature Review

The economics literature, as well as canonical models of international trade presented in graduate courses, most frequently model transfers of tariff revenue as a lump-sum payment to *consumers*, rather than firms. These payments appear on the right-hand side of consumers' budget constraints, increasing the amount of money they have available to spend. Academic papers that use such a feature within their model including Alessandria et al. (2025), Bagwell and Lee (2020), and Lashkaripour and Lugovskyy (2019); with the exception of Alessandria et al. (2025), tariff revenue distribution is one of many features used in these papers' modeling and not the main focus of analysis. Jafarey and Lahiri (2024) and Lahiri and Nasim (2006) do feature payments to firms as the primary focus of their analysis, but limit their discussion to rebates for tariffs on intermediate goods, as a way to compensate firms that may sell to a domestic market but may have to pay tariffs on intermediate inputs that they import from abroad. This paper discusses a general structure for modeling lump-sum tariff payments in a wide variety of situations.

This paper also contributes to a series of working papers by Commission staff that embed heterogeneous-productivity firms within a standard partial equilibrium model. Barbe, Chambers, Khachaturian, and Riker (2017) and Mueller and Riker (2020) discuss this embedding process more generally, and Khachaturian and Riker (2016) uses a heterogeneous-productivity partial equilibrium model in order to analyze the effect of decreased fixed costs on international trade in services. The modeling framework in this paper uses heterogeneous productivity in firms to quantify the effects of tariff transfer payments on firms of varying productivities, providing an estimate for the proportion of firms that would otherwise be forced to exit the market but are able to remain operational due to the lump-sum transfer they receive.

More broadly, this paper complements a wider array of academic literature on trade with heterogeneous firms. The seminal paper in this body of scholarship is Melitz (2003), the first paper that used a Pareto firm productivity distribution to model the effects of trade liberalization. Helpman, Melitz, and Yeaple (2004) build on this framework by using a model with Pareto-distributed productivities to explain how firms decide between exporting to another country and increasing foreign direct investment in their home country. Demidova (2008), Demidova and Rodriguez-Clare (2009), and Felbermayr, Jung, and Larch (2013) make further theoretical contributions to general equilibrium modeling with Pareto-distributed productivities, while Spearot (2016) and Melitz and Redding (2015) calibrate heterogeneous-productivity firm models using data inputs. While this paper does not present any data-driven results, it does discuss in theory how its model could be calibrated, as well as what data inputs would play a role in this calibration process.

2 Model

Because this model is a partial equilibrium model, each industry has its own separate set of equilibrium conditions, and in the case of multiple industries the model assumes that prices and consumption within the various industries do not influence one another. Since industries for the most part are separate from one another, the equations in this section do not index variables by industry.[2]

2.1 Consumers

Goods in this model come from any one of N origin countries, indexed by j. The model further subdivides consumption from each origin according to varieties z, and consumers in country i choose among origins and varieties to maximize the following CES utility function:

$$\left(\sum_{j=1}^{N} \int_{0}^{m_{ji}} c_{ji}(z)^{\frac{\sigma_i - 1}{\sigma_i}} dz\right)^{\frac{\sigma_i}{\sigma_i - 1}}$$

Consumers in country i substitute with elasticity σ_i among goods of different varieties and from different origin countries. m_{ji} is an equilibrium allocation representing the total measure of goods that are exported from j to i; a higher m_{ji} indicates that country j is sending a higher volume of shipments to country i.

Let E_i denote total consumer expenditure on a given industry in country i. As is common in partial equilibrium models,[3] I assume that $E_i = Y_i P_i^{-\theta}$, $\theta = 1$, and that Y_i does not change in any counterfactual situations. The consumer's first-order conditions lead to the following equation:

$$E_{ji}(z) = Y_i P_i^{\sigma_i - 1} P_{ji}(z)^{1-\sigma_i} \;\forall z \tag{1}$$

[2] The distribution of tariff revenue discussed in section 3 may have cross-industry effects, depending on how it is performed. In discussing modeling scenarios with cross-industry payments, industry-level indexing would become necessary.

[3] See Riker and Hallren (2017).

where P_i is the aggregate price index for country i and $P_{ji}(z)$ is the price of variety z when shipped from country j to country i. The aggregate price index P_i is given by

$$P_i^{1-\sigma_i} = \sum_{j=1}^{N} \int_0^{m_{ji}} P_{ji}(z)^{1-\sigma_i} dz$$

2.2 Firms

Goods are produced by a continuum of firms, each with productivity x. I assume, without loss of generality, that each variety z maps to a productivity x, and I accordingly use the notation $x(z)$. A firm in country j exporting variety z to country i maximizes the following profit function:

$$p_{ji}(z)q_{ji}(z) - \frac{w_j \tau_{ji} q_{ji}(z)}{x(z)} - w_j f_{ji}$$

In other words, the firm earns revenue from selling output $q_{ji}(z)$ at price $p_{ji}(z)$, while paying variable costs $\frac{w_j \tau_{ji} q_{ji}(z)}{x(z)}$ and fixed costs $w_j f_{ji}$. A higher productivity $x(z)$ lowers the cost of producing z, while a higher tariff τ_{ji} raises the cost of sending variety z from j to i with $\tau_{jj} = 1$. f_{ji}, the fixed cost of production, varies based on the origin country and destination country. Finally, w_j is the unit cost of producing the good; if labor is the only input in production, then w_j would simply equal the wage. I take these production inputs to be the numeraire good, and so w_j is normalized to 1; since w_j always equals 1, it will not appear in any future equations.

Market clearing indicates that $q_{ji}(z) = c_{ji}(z)\ \forall z$. Using (1) to replace $c_{ji}(z)$ in the firm's profit function and taking first-order conditions,[4] I find that

$$p_{ji}(z) = \frac{\sigma_i}{\sigma_i - 1} \frac{\tau_{ji}}{x(z)}$$

where $\frac{\sigma_i}{\sigma_i - 1}$ is the firm's markup over marginal cost $\frac{\tau_{ji}}{x(z)}$.

2.3 Productivity Distribution

Firm productivity x in country j follows the Pareto distribution

$$G_j(x) = 1 - x^{-\gamma_j}$$

[4]The appendix goes into all mathematical derivations in greater detail.

which has the density function

$$\frac{dG_j(x)}{dx} = \gamma_j x^{-\gamma_j - 1}$$

I refer to γ_j as the 'shape parameter', and a higher value of γ_j indicates that the distribution of firms in country j will be more clustered toward lower productivities.[5]

The properties of the Pareto distribution both make the model more tractable and ensure the existence of a cutoff productivity, $\bar{x}_{ji}$, for all destination-origin pairs ji such that all j firms with productivities higher than $\bar{x}_{ji}$ export to i and all j firms with productivities lower than $\bar{x}_{ji}$ do not export to i because they cannot make enough operating profit to recoup their fixed costs. $\bar{x}_{ji}$ depends on fixed costs f_{ji}, tariff rates τ_{ji}, and prices, and differs by origin and destination. Given the differing levels of profitability along each route, a firm with a given productivity may choose to sell goods within its own domestic market without exporting, and a firm with a given productivity may choose to export to one destination country but not another.

Let μ_j represent the measure of *potential* firms in country j. The measure of firms in j able to export to i is then given by

$$m_{ji} = \mu_j \int_{\bar{x}_{ji}}^{\infty} dG_j(x) = \mu_j \bar{x}_{ji}^{-\gamma_j}$$

where $\bar{x}_{ji}^{-\gamma_j}$ is the fraction of total firms in j with sufficiently high productivities to export to i.

2.4 Equilibrium Equations

Country i's expenditure on goods from country j, or alternatively the value of imports from j to i, is given by

$$\begin{aligned} E_{ji} &= \mu_j \int_{\bar{x}_{ji}}^{\infty} c_{ji}(x) p_{ji}(x) dG_j(x) \\ &= \mu_j E_i P_i^{\sigma_i - 1} \Big(\frac{\tau_{ji} w_j \sigma_i}{(\sigma_i - 1)} \Big)^{1-\sigma_i} \frac{\bar{x}_{ji}^{\sigma_i - 1 - \gamma_j}}{\gamma_j - (\sigma_i - 1)} \end{aligned} \tag{2}$$

[5]Note that both the elasticity of substitution and shape parameter are allowed to differ by country. If modelers do not have enough data to estimate γ_j and σ_i separately for every country, then can assume $\gamma_j = \gamma$ and $\sigma_i = \sigma$ for all j and i.

This last step requires an assumption that $\gamma_j \geq \sigma_i - 1$ for consumption to be non-negative. Likewise, the aggregate price in country i is given by

$$\begin{aligned} P_i^{1-\sigma_i} &= \sum_{j=1}^{N} \left[\mu_j \int_{\overline{x}_{ji}}^{\infty} p_{ji}(x)^{1-\sigma_i} dG_j(x)\right] \\ &= \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \sum_{j=1}^{N} \mu_j (\tau_{ji})^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - \gamma_j - 1} \end{aligned} \tag{3}$$

An equilibrium is characterized by consumption expression (2), price index expression (3), and finally zero-profit conditions that use the definition of $\overline{x}_{ji}$ as the boundary point where a country j firm is indifferent between selling in country i and not selling in country i. In other words:

$$\begin{aligned} f_{ji} &= \pi(\overline{x}_{ji}) \\ &= p_{ji}(\overline{x}_{ji}) c_{ji}(\overline{x}_{ji}) - \frac{\tau_{ji} c_{ji}(\overline{x}_{ji})}{\overline{x}_{ji}} \end{aligned}$$

Substituting for $p_{ji}(\overline{x}_{ji})$ and $c_{ji}(\overline{x}_{ji})$ and doing some algebra, I get

$$f_{ji} = Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \frac{1}{\sigma_i} \tag{4}$$

Equations (2)-(4) are sufficient to solve for an equilibrium of this model without transfer payments, given values for $\gamma_j, \sigma_i, f_{ji}, \tau_{ji}$ and μ_j.

3 Distribution of Tariff Revenue

A higher value of τ_{ji} raises the minimum productivity necessary to export from j to i, and some firms would exit that market. The exodus of exporting firms would eliminate some of the competition faced by domestic firms producing and selling goods in i, lowering the domestic productivity threshold $\bar{x}_{ii}$. However, if other countries also maintained high tariffs, domestic producers in i would face a higher export barrier $\bar{x}_{ij}$. This potential loss in exports could decrease industry profits, while an increase in domestic sales could increase profits. The distribution of tariff revenue as lump-sum transfers introduces an additional benefit of higher tariffs for domestic industries, and can compensate those industries for the negative effects that they may incur from higher foreign tariffs. While Section 2 went through the mechanics of a partial equilibrium model as described in Barbe et al. (2017) and Mueller and Riker (2020), this section describes a novel methodology for introducing tariff transfer payments within this partial equilibrium modeling framework.

The precise formula for distributing tariff revenue is both hypothetical and unknown. Firms could be paid out the revenue from tariffs imposed on imports of their own industry, or alternatively paid according to some share of tariff revenue collected across the entire economy. Furthermore, firms may not know they are receiving transfer payments until after they have already committed to a production and pricing decision, or alternatively they receive notice of such transfers in advance and consider their expected payments when determining production. This paper therefore discusses multiple possible ways to model the distribution of tariff revenue, taking into account that transfer payments could be collected and/or distributed differently.

3.1 Revenue Distribution

I begin with a note clarifying the distinction between total tariff revenue collected across the entire economy, and the amount of tariff revenue received by each firm. The two values are not equivalent, because the economy consists of more than one firm. I set R_j to be the total tariff revenue collected across the entire economy of country j, and r_j to be the transfer payment received by each firm within country j.

As discussed in Section 2, μ_j gives the total measure of firms in a country j. This parameter μ_j represents the total amount of firms that *could* produce in country j, and does not change in any counterfactual situations. The total measure of firms in country j that actually produce for market i is given by $\mu_j x_{ji}^{-\gamma_j}$ and the total measure of firms in j that produce for any market at all, m_j, is this expression summed across all possible destinations, or $\mu_j \sum_{i=1}^{N} \bar{x}_{ji}^{-\gamma_j}$. In all future discussions, I will refer to this measure by the more concise name of 'firm participation.'

Assuming that all firms within a country receive the same amount, R_j and r_j are related by the following identity:

$$r_j = \frac{R_j}{\mu_j \sum_{i=1}^{N} \bar{x}_{ji}^{-\gamma_j}}$$

The relationship between the two quantities depends on μ_j and γ_j, both model parameters, and $\{\bar{x}_{ji}\}_{i=1}^{N}$, a set of equilibrium allocations.

3.2 Payment Values

If transfer payments are industry-specific, then domestic firms receive a share of the revenue collected from tariffs in that industry. For example, apple producers would receive transfer payments funded by tariffs on imported apples. One implication of this policy is that industries with higher volumes of foreign imports would receive higher payments, as more tariff revenue would be collected from those industries. Conversely, industries with high export demand but low import demand would not benefit to any great degree from the distribution of tariff revenue.

Total transfer payments would be given by the expression

$$R_i = \psi_i \sum_{j \neq i}^{N} (\tau_{ji} - 1) \int_{\bar{x}_{ji}}^{\infty} E_{ji}(x) dG_j(x)$$

as imports from all foreign sources and of all productivities would be subject to the tariff rate $\tau_{ji} - 1$.[6] The ψ_i parameter ranges from 0 to 1, allowing modelers to vary the magnitude of the payment rather than limit themselves to situations where firms either receive no transfer payments or receive transfers equal to the entirety of the eollected tariff revenue.

Alternatively, each industry could receive some share of total tariff revenue collected across the entire economy. Since a discussion of this formulation entails distinguishing between different industries, in the following equations I will index industries by the letter n.

Total tariff revenue across the entire economy in country i is given by

$$R_{in} = s_n \sum_{n=1}^{K} \sum_{j \neq i}^{N} (\tau_{jin} - 1) \int_{\bar{x}_{jin}}^{\infty} E_{jin}(x) dG_{jn}(x),$$

summing the industry-specific revenue in Section 3.1.1 across all industries n. s_n refers to industry n's share of the total tariff revenue available in country i, and a calculation of R_{in} in this setup requires some assumption about what those shares might be. For example, a country might want to compensate industries more adversely affected by foreign tariffs, in which case s_n could reflect industry n's share of total exports from country i.

$$R_{in} = \frac{E_{in}}{\sum_{n=1}^{K} E_{in}} \psi_i \sum_{n=1}^{K} \sum_{j \neq i}^{N} (\tau_{jin} - 1) \int_{\bar{x}_{jin}}^{\infty} E_{jin}(x) dG_{jn}(x),$$

Alternatively, a country might want to encourage production for the domestic market, in which case s_n could reflect industry n's share of i's share of domestic production for the domestic market:

$$R_{in} = \frac{E_{iin}}{\sum_{n=1}^{K} E_{iin}} \psi_i \sum_{n=1}^{K} \sum_{j \neq i}^{N} (\tau_{jin} - 1) \int_{\bar{x}_{jin}}^{\infty} E_{jin}(x) dG_{jn}(x),$$

[6] The firm's profit function in Section 2.2 defined τ_{ji} as greater than 1, or 1 plus the imposed tariff rate. To keep notation consistent, the formula in this section contains $\tau_{ji} - 1$.

3.3 Unexpected vs. Expected Payments

If transfer payments are unexpected, firms do not consider them when choosing production quantities, and the payments simply arrive after an equilibrium has already been determined. Since firms have already made production decisions, these unexpected payments does not help additional firms to enter the domestic market, but they do alleviate profit losses that domestic firms may incur due to the imposition of foreign tariffs.

Researchers may also want to consider a scenario where firms know they are receiving some sort of tariff revenue transfer, and take this transfer payment into account when deciding how much to produce. I assume that while domestic industries expect a payment, their production decisions do *not* consider how payments depend linearly on values of imports from other countries. Such consideration would introduce a counter-intuitive mechanism whereby domestic industries have an incentive to reduce their production in order to stimulate foreign imports, which would in turn increase the tariff revenue from which domestic industries benefit. I consider such an effect to be unrealistic and unlikely, and instead model 'expected' transfer payments as a quantity r_{jn} that enters on the left-hand side of each industry's zero-profit condition. The equilibrium in this situation would be the solution to a fixed-point problem where R_{in} is equal to some function g of equilibrium imports and industry-specific tariff rates $\tau_{jin} - 1$.

Equations (2) and (3) would be unchanged in the case of expected transfers, while (4) would add the payments to its left-hand side. The resultant equilibrium would be the solution to the equations

$$
\begin{aligned}
E_{ji} &= \mu_j E_i P_i^{\sigma_i - 1} \left(\frac{\tau_{ji} w_j \sigma_i}{(\sigma_i - 1)}\right)^{1-\sigma_i} \frac{\overline{x}_{ji}^{\sigma_i - 1 - \gamma_j}}{\gamma_j - (\sigma_i - 1)} \\
P_i^{1-\sigma_i} &= \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \sum_{j=1}^{N} \mu_j (\tau_{ji})^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - \gamma_j - 1} \\
f_{ji} - r_j &= Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \frac{1}{\sigma_i} \\
R_j &= g(\{\tau_{nj}\}_{n=1}^{N}, \{E_{nj}\}_{n=1}^{N})
\end{aligned}
$$

Figure 1 uses a timeline format to review the setup discussed in Sections 2 and 3. In the absence of transfer payments, each firm that could potentially produce in the economy finds out its productivity parameter, then determines how much output to produce for each destination based on how this parameter compares to the minimum productivity needed to generate positive profits along that route. If payments are expected, then firms will adjust production decisions accordingly. If the payments come unexpectedly, then firms only find out about them after deciding how much output to produce for each destination, so they only affect final profit levels.

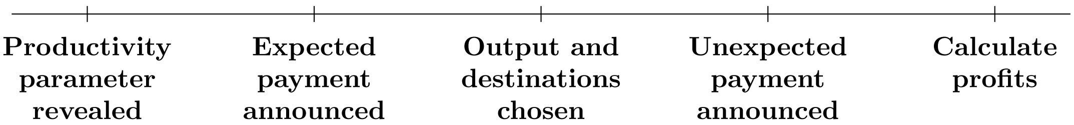


Figure 1: Model Timeline

4 Calibration and Estimation

4.1 Calibration

The model as described in Section 2 and Section 3 features five external parameters: $\gamma_j, \sigma_i, \tau_{ji}, f_{ji}$ and μ_j. Of these parameters, γ_j, σ_i, and τ_{ji} can be directly estimated from data. The estimation of τ_{ji} is particularly straightforward as the USITC's DataWeb provides tariff rates at the Harmonized Tariff System 8-digit level. In the event that a researcher wants to analyze a broader sector or industry, they can aggregate the tariff rates for that industry's constituent HTS 8-digit subheadings using either a simple average or weighting the tariff rates by HTS8-level imports.

Previous papers have discussed methods for calibrating σ_i, the elasticity of substitution between goods of different origins, and γ_j, the shape parameter that determines the distribution of firm productivities. The two main methods used by USITC economists for estimating σ_i are the markup method, as introduced in Ahmad and Riker (2019), and the trade cost method, as discussed in Riker (2019).[7] While the markup method may be more appropriate in models or papers where firm markups are a primary object of interest, the trade cost method estimates elasticities at a more granular product level; in addition, the data used as inputs in the trade cost method has been updated more frequently than the data used as inputs in the markup method.

The Pareto shape parameter γ appears much less frequently in economic modeling than the CES elasticity, and the literature discussing how to estimate this parameter is accordingly more sparse. Ahmad and Akgul (2019) introduce a maximum likelihood method for identifying Pareto shape parameters, and Ahmad and Akgul (2017) describe a visual method for finding γ that involves producing a log-log plot of firm productivity and its lower bound. di Giovanni, Levchenko and Rancière (2011) and Chaney (2008) estimate γ with regression-based methods using firm-level data.

In addition to parameters γ_j, σ_i and τ_{ji}, whose estimation I have just discussed, fixed costs f_{ji} and firm measures μ_j are necessary to solve for an equilibrium. However, f_{ji} and μ_j do not have any clear counterpart in the data, nor are there values for these parameters that I could impose in theoretical simulations of the model such as the ones performed in this paper. I will therefore discuss how to use alternative model

[7] Feenstra et al. (2018) uses a Melitz-style model with CES preferences to estimate elasticities of substitution, and discusses other approaches used in the academic literature to perform this estimation.

outputs that do exist in the data, namely consumption expenditure and the share of firms that export to each destination.

Sources such as the USITC's DataWeb or UN Comtrade document industry-specific exports from one country to another. While direct measures are typically not available for consumption expenditure on domestically produced goods, such measures can generally be imputed.[8] Data measures can therefore represent model quantities E_{ji} and Y_i in a baseline (pre-counterfactual) equilibrium, and I use the symbol β_{ji} to represent an expenditure share, i.e. $\beta_{ji} = \frac{E_{ji}}{Y_i}$.

I use the symbol ϕ_{ji} to denote the share of domestically producing firms in j that ship to destination i. This measure typically cannot be computed without detailed firm-level data.[9] However, even in the absence of such detailed data modelers can input plausible values of the fraction of firms that export to each destination, and in a two-country model $\phi_{ji},\ j \neq i$ simply represents the share of firms in j that export at all.[10] Using properties of the Pareto distribution, we know

$$\begin{aligned} \phi_{ji} &= \frac{1 - G_j(\bar{x}_{ji})}{1 - G_j(\bar{x}_{jj})} \\ &= \big(\frac{\bar{x}_{ji}}{\bar{x}_{jj}}\big)^{-\gamma_j} \end{aligned}$$

so it is a function of the ratio between the productivity cutoff necessary to produce at all and the productivity cutoff necessary to export to destination j.

4.2 Solving for an Equilibrium

This subsection discusses how to solve the model for equilibrium changes in a counterfactual with different tariff rates. While the rest of the paper focuses on this counterfactual, the solution methodology could easily be adapted to analyze other counterfactual situations. For example, the modeling framework can be used to examine a situation where fixed costs change and tariff rates do not, or where tariffs and fixed costs both change.

Solving for an equilibrium requires rearranging equations (2)-(4) so that they are in terms of changes in allocations rather than the levels of the allocations themselves. Let $\hat{y}$ represent the counterfactual level of a variable y, so that $\hat{\tau}_{ji}$ represents counterfactual tariff levels and τ_{ji} represents baseline tariffs. I will start by explaining how to solve for an equilibrium without any distribution of tariff revenue, then explain how the

[8]The ITC's ITPD database is one source containing such consumption estimations.

[9]Papers that use detailed firm-level datasets include Aghion et al. (2022); Mayer, Melitz and Ottaviano (2014); Chaney (2008); and Eaton, Kortum, and Kramarz (2004a, 2004b). While papers with firm-level data more frequently use European datasets, Keller and Yeaple (2009) and Harrigan, Ma, and Shlychkov (2015) are two papers that present findings using firm-level data from the United States.

[10]Bernard et al. (2007) find that around 14 percent of all firms in the U.S. export. The International Trade Administration's Exporters Database also provides information on U.S. firms that export.

addition of these transfers into the model changes the solution methodology.

In an equilibrium without transfer payments, fixed costs do not change in the counterfactual, so $\hat{f}_{ji} = f_{ji}$. Because of this relationship, I know that the right-hand side of equation (4) in any counterfactual scenario will always be equal to the right-hand side of equation (4) in the baseline equilibrium. Some algebraic rearrangement gives

$$\frac{\hat{\bar{x}}_{ji}}{\bar{x}_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})(\frac{\hat{P}_i}{P_i})^{-1} \tag{5}$$

Likewise, dividing the LHS and RHS of consumption equation (2) in the counterfactual by their counterparts in the baseline gives

$$\frac{\hat{E}_{ji}}{E_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})^{1-\sigma_i}(\frac{\hat{P}_i}{P_i})^{\sigma_i-1}(\frac{\hat{x}_{ji}}{x_{ji}})^{\sigma_i-1-\gamma_j} \tag{6}$$

An important, and useful feature of this setup is that μ_j drops out of the equation since it does not vary by time.

Price index equation (3) is more difficult because variables enter into it additively rather than multiplicatively. The rewriting of (3) comes from moving $P_i^{1-\sigma_i}$ over to the right-hand side of the equation, then using (5) and (2) to replace $\mu_j x_{ji}^{\sigma_i-\gamma_j-1} P_i^{\sigma_i-1}$ with β_{ji} and equilibrium ratios. Doing so eventually yields

$$1 = \sum_{j=1}^{N} \beta_{ji}(\frac{\hat{P}_i}{P_i})^{\gamma_j}(\frac{\hat{\tau}_{ji}}{\tau_{ji}})^{-\gamma_j} \tag{7}$$

In an equilibrium without revenue distribution, I am therefore able to solve for counterfactual percent changes using calibrated values of σ_i and γ_j as well as data on consumer expenditure. Note that I am able to solve for counterfactual *levels* of consumer expenditure, and not just percent changes, because I know E_{ji}, so given E_{ji} and the ratio $\frac{\hat{E}_{ji}}{E_{ji}}$ I can get an absolute level of counterfactual consumption.

With firms receiving lump-sum payments, the situation changes slightly. Lump-sum payments effectively alter the fixed costs for firms to enter the market, so (5) is no longer true. Instead I have that

$$\frac{\hat{\bar{x}}_{ji}}{\bar{x}_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})(\frac{\hat{P}_i}{P_i})^{-1}(\frac{\hat{f}_{ji}}{f_{ji}})^{\frac{1}{\sigma_i-1}} \tag{8}$$

where $\hat{f}_{ji} = f_{ji} - r_j$. Unfortunately, I do not observe levels of fixed costs f_{ji}. I outline a method for finding this new equilibrium using ϕ_{ji}, the fraction of domestically producing firms in j that export to i.

I rewrite the ratio $\frac{\hat{f}_{ji}}{f_{ji}}$ as

$$1 - \frac{r_j}{f_{ji}}$$

In the event that transfer payments are unexpected, I observe quantity R_j (it is equivalent to total tariff revenue), but not r_j, because I don't know the total quantity of firms. Each individual firm would face a change in fixed costs of $1 - \frac{r_j}{f_{ji}}$. I correspondingly define $F_{ji} = f_{ji} m_{ji}$ as the total fixed cost amount paid by all firms using the route from j to i. This total fixed cost quantity is

$$F_{ji} = Y_i \frac{(\gamma_j - (\sigma_i - 1))\beta_{ji}}{\gamma_j \sigma_i}$$

and the counterfactual change in fixed costs[11] is

$$1 - \frac{r_j}{f_{ji}} = \frac{R_j}{F_{ji}} \Big(\sum_k \big(\frac{\phi_{jk}}{\phi_{ji}} \big) \big(\frac{\hat{m}_{jk}}{m_{jk}} \big) \Big)^{-1} \tag{9}$$

Remember that the ratio $\frac{\hat{m}_{jk}}{m_{jk}}$ is equal to $\big(\frac{\hat{x}_{jk}}{x_{jk}}\big)^{-\gamma_j}$. Once that substitution is made, the ratio $\frac{r_j}{F_{ji}}$ can be written in terms of known quantities and counterfactual ratios.

In the event that transfers arrive unexpectedly, I solve for an equilibrium with (6)-(8), then use the ratios $\frac{\hat{m}_{jk}}{m_{jk}}$ to find the change in fixed costs for each origin-destination pair. Applying this change in fixed costs, I am able to find counterfactual changes in profits and the size of the industry.

If firms expect payments in advance, then equations (5)-(7) change slightly. (5) becomes (8), and (7) becomes

$$1 = \sum_{j=1}^{N} \beta_{ji} \big(\frac{\hat{P}_i}{P_i} \big)^{\gamma_j} \big(\frac{\hat{\tau}_{ji}}{\tau_{ji}} \big)^{-\gamma_j} \big(\frac{\hat{f}_{ji}}{f_{ji}} \big)^{1 - \frac{\gamma_j}{\sigma_i - 1}} \tag{10}$$

The equilibrium then consists of (6), (8), and (10).

Finally, I present a formula for the change in country j's total firm participation:

$$\frac{\hat{m}_j}{m_j} = \frac{\sum_{i=1}^{N} \phi_{ji} \big(\frac{\hat{\overline{x}}_{ji}}{\overline{x}_{ji}} \big)}{\sum_{i=1}^{N} \phi_{ji}} \tag{11}$$

The estimation of $\frac{\hat{m}_j}{m_j}$ would not be possible without knowledge of $\{\phi_{ji}\}_{i=1}^{N}$.

[11] For the complete algebraic derivation, see the appendix.

5 Simulated Results

In this section, I present results of simulating the model after selecting a series of hypothetical values for the parameter inputs. Because simulation results are theoretical, they are intended to showcase the model's capabilities rather than provide commentary on specific policies or countries.

I begin with the simplest possible scenario in which I run simulations with one industry and two countries that are identical in every possible way. Bilateral tariffs are symmetric, and increase from 5 percent in the baseline version of the model to 25 percent in the counterfactual. I then illustrate additional situations where the countries differ slightly from one another, or alternatively are identical but conduct trade with a third country that is unaffected by the increase in tariffs. Finally, I run simulations where the two countries conduct trade in more than one industry.

5.1 Identical Countries

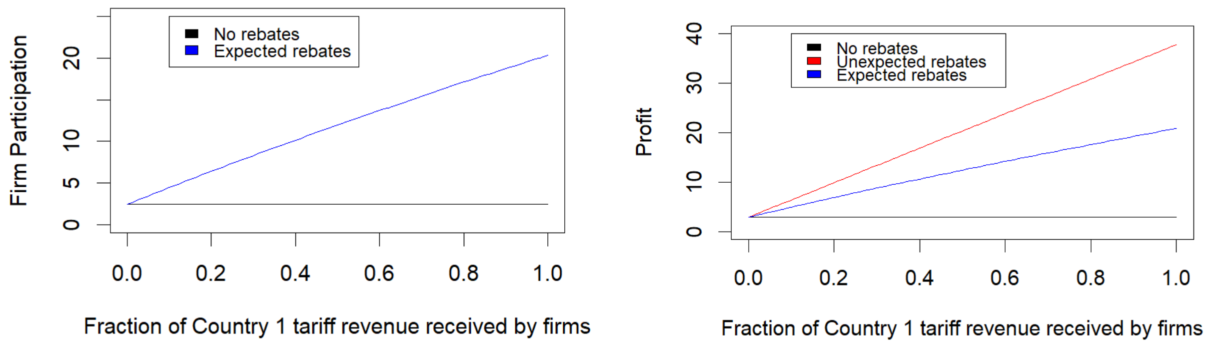
Initial simulations feature two identical countries, which both have shape parameter $\gamma_j = 4$ and elasticity of substitution $\sigma_i = 3$. Furthermore, both countries have aggregate expenditure $Y = 100$, and spend 70% of their total expenditure on domestically produced goods compared with 30% on imported goods.

Finally, I assume that 20 percent of domestically producing Country 1 firms also export to Country 2 ($\phi_{12} = .2$). The results that I present will focus on Country 1, and I do not need to set a value for ϕ_{21} since providing ϕ values for Country 2 is not necessary to compute counterfactual changes in Country 1.

Figure 2 shows the results of simulating an increase in tariffs on trade between Country 1 and Country 2. I display the results of varying ψ, which governs the size of the transfer payments firms get; if $\psi = 1$, then Country 1 firms would receive all of the revenue generated by imposing tariffs on Country 2 imports. Figure 2a) displays the percent change in total firm participation, $\frac{\hat{m}_1}{m_1} - 1$, and Figure 2b) displays the percent change in total country 1 profits, $\frac{\hat{\pi}_1}{\pi_1} - 1$.

As shown in Figure 2, firm participation increases in ψ at a roughly linear rate in the scenario where firms expect to receive payments, while profits increase in ψ in a roughly linear fashion under both scenarios. If ψ is set at .5, meaning that firms receive half of the maximum possible payment value, firm participation would increase by about 23.9% in the expected case, while profits would increase by 26.4% in the unexpected case and 14.4% in the expected case.

Table 1 provides the results of simulating an increase in tariffs when I choose ψ so that the transfer given to Country 1 firms equals the total tariff fees those firms must pay when exporting to Country 2. Rows describe Country 1 equilibrium allocations of interest, while columns respectively provide counterfactual changes in a scenario with no transfer payments, a scenario with unexpected transfer payments, and a



(a) Change in firm participation

(b) Change in total profit

Figure 2: Counterfactual outcomes as a function of payment sizer

scenario with expected transfer payments.

Allocation	No payments	Unexpected payments	Expected payments
Percent change in firm participation	7.87%	7.87%	37.1%
Percent change in domestic production	17.7%	17.7%	21.8%
Absolute change in domestic production	12.4	12.4	15.3
Percent change in imports from Country 2	-42.4%	-42.4%	-50.5%
Absolute change in imports from Country 2	-12.4	-12.4	-15.3
Percent change in total profits	-0.01%	52.7%	26.4%
Absolute change in total profits	-1.87×10^{-3}	8.79	4.40

Table 1: Model simulation results for identical countries

When firms expect payments to arrive, they know that their fixed costs will be lower. This decrease in barriers to entry lowers the productivity cutoff $\overline{x}$ and allows a greater number of firms to produce in Country 1. If the payments are unexpected, then they do not enable additional firms to produce in Country 1, but they do contribute to profits of Country 1 firms that are already operating.

Without transfer payments, an increase in the tariff rate between Countries 1 and 2 causes total firm participation to increase by 7.79%, as an influx of firms that produce for the domestic market outweighs an exodus of firms no longer able to export to Country 2. Profits are essentially unchanged because, since the countries are identical in every way, the increase in revenue from domestic sales is equal to the decrease in revenue from exporting. The introduction of unexpected transfers substantially benefits firms, with a tariff change leading to a 52.7% increase in profits. In the simulation with expected transfers, the increase in firm profits is lower than in the simulation with unexpected transfers, but the increase in domestic production is higher. While a greater number of operating firms increases production within Country 1, the larger number of less productive firms present in the market undercuts market power and lowers profits compared to a situation where those additional firms do not enter.

5.2 Non-Identical Countries

I now consider two alternate scenarios where the countries are not identical. In the first scenario, the countries no longer have identical initial expenditure shares; in particular, Country 1 spends only 20% of its total expenditure on imports from Country 2, while Country 2 continues to spend 30% of its expenditure on imports from Country 1. In the second scenario, I alter the relative sizes of each country's economy by increasing total expenditure in Country 1 to 200, while total expenditure in Country 2 remains at 100.

Table 2 displays results from these scenarios, with all rows showing counterfactual changes or counterfactual percent changes. In these simulations and in all future simulations, I continue to set ψ so that Country 1 firms do not receive more than the value of the tariffs that they pay.

Allocation	$\beta_1 = (.8, .2)$			$Y_1 = 200$		
	None	Unexpected	Expected	None	Unexpected	Expected
Firm participation	2.41%	2.41%	18.9%	7.87%	114%	39.1%
Domestic production	11.1%	11.1%	12.8%	17.7%	17.7%	21.8%
Domestic production (absolute)	8.94	8.94	10.2	28.4	28.4	34.9
Country 2 imports	-44.7%	-44.7%	-51.1%	-42.4%	-42.4%	-50.9%
Country 2 imports (absolute)	-8.93	-8.93	-10.2	-16.6	-16.6	-20.3
Total profits	-3.16%	27.0%	12.3%	7.29%	69.3%	38.4%
Profits (absolute)	-0.58	4.95	2.26	0.909	12.0	6.60

Table 2: Model simulation results with non-identical countries

As shown in Table 2, if domestic production is given a higher weight in initial expenditure (columns 2-4), counterfactual increases in firm participation and total profits are lower compared to their values in Table 1. Profits in the absence of transfer payments now decrease by 3.16%, while unexpected transfers cause profits to increase by 27% and expected transfers cause profits to increase by 12.3%. The boost to domestic production provided by tariffs is less valuable if Country 1 already had a higher weight placed on domestic production, and transfers are likewise less beneficial for Country 1 firms because Country 1 has a lower import value from which to generate tariff revenue. Meanwhile, a decrease in international trade more adversely affects Country 1 firms than it does in Table 1, because Country 2 is now more export-dependent.

An increase in the size of Country 1's economy relative to Country 2's economy (columns 5-7) results in greater increases in firm participation and total profits compared to the case where both countries are identical. Even though percent changes in domestic production and imports are the same as they were in Table 1, tariffs are now re-orienting Country 1 firms toward a domestic market that is larger and therefore offers a higher revenue base. When transfer payments are expected, firms further internalize the greater size of Country 1's economy, resulting in greater increases in domestic production compared to Table 1.

5.3 Simulation with Three Countries

I next simulate counterfactual results in a setup where Country 1 and Country 2 are once again identical but also trade with a third country, Country 3. Instead of allocating 30% of initial consumer expenditure to Country 2 imports, Country 1 now allocates 20% to imports from Country 2 and 10% to imports from Country 3, while Country 2 allocates 20% to imports from Country 1 and 10% to imports from Country 3. Country 3, meanwhile, is more import-focused, with 25% of its expenditure going to imports from Country 1 and 25% also going to imports from Country 2. Country 3 otherwise has all the same parameters as Countries 1 and 2, and $\phi_{13} = .2$ so that 20% of firms producing in Country 1 export to Country 3, just as 20% of firms producing in Country 1 export to Country 2.

Crucially, while Country 1 and Country 2 increase tariffs on each other, tariffs on imports from Country 3 remain at 5%. Table 3 shows the results of this counterfactual, with ψ set so that firms receive transfers according to the total value of foreign tariffs that they pay.

Allocation	No transfers	Unexpected transfers	Expected transfers
Percent change in firm participation	1.60%	1.60%	22.5%
Percent change in domestic production	11.2%	11.2%	14.9%
Absolute change in domestic production	7.82	7.82	10.4
Percent change in imports from Country 2	-44.7%	-44.7%	-52.2%
Absolute change in imports from Country 2	-8.93	-8.93	-10.2
Percent change in total profits	-0.966%	51.5%	34.3%
Absolute change in total profits	-0.185	9.02	6.57

Table 3: Results for simulations featuring a third country

When the model features a third country whose tariff rates are unchanged, Country 1 industries do not benefit as much from the tariff increase on Country 2. The percent changes in firm participation and domestic production in Table 3 are smaller for all three simulations than the corresponding numbers in Table 1. In addition, the adverse impact of tariffs on imports from Country 2 is stronger (more negative) in Table 3 than in Table 1 for a similar reason; as tariffs make Country 2 imports less desirable, Country 1 consumers have another location to which they can shift their consumption. The addition of Country 3 does not make an enormous difference to counterfactual changes, however, as most results differ from Table 1 by 1-2 percentage points.

Percent changes in profits compared to Table 1 are more negative in the case of no transfer payments and less positive in the case of unexpected transfer payments, as Country 3 absorbs some of the benefits that would go to Country 1 producers. However, in the case where transfer payments are expected, profit increases are actually *higher* than in Table 1. The increase in tariffs on Country 2 results in higher exports from Country 1 to Country 3, and when firms are aware of a decrease in fixed costs they can take advantage of this shift in export destinations more effectively, overcoming the loss in revenue from exports to Country 2.[12]

5.4 Tariff Transfer Payment Simulation with Industry Weights

Finally, I solve this model for counterfactual changes in a scenario where tariff revenue is allocated among different industries, rather than each industry receiving the revenue generated from imports in that industry. I consider two new counterfactuals: one in which tariff revenue is distributed based on a given industry's share of domestic revenue (to incentivize domestic production) and one in which tariff revenue is distributed based on a given industry's share of country 1's exports (to compensate industries that are more adversely affected by foreign tariffs). The two industries differ only in their initial consumer expenditure shares, with Industry 1 identical to the industry in the previous section and Industry 2 having the following matrix:

	Country 1	Country 2
Country 1	0.6	0.5
Country 2	0.4	0.5

Table 4: Expenditure shares for Industry 2 in multi-industry scenario

Compared to Industry 1, Industry 2 is more import-dependent in terms of consumption for country 1, as well as more export-facing for firms in country 1. Table 4 displays the results of comparing industry-weighted results for Industry 1 with the unexpected and expected results for Industry 1 displayed in Table 1.

Allocation	Unexpected, industry-only	Unexpected, domestic	Unexpected, export	Expected, industry-only	Expected, domestic	Expected, export
Firm participation	52.7%	52.7%	52.7%	37.1%	44.9%	33.7%
Domestic production	17.7%	17.7%	17.7%	21.8%	22.7%	21.4%
Country 2 Imports	-42.4%	-42.4%	-42.4%	-50.5%	-53.0%	-50.0%
Total profits	52.7%	68.6%	47.8%	26.4%	33.2%	23.3%

Table 5: Counterfactual percent changes for different payment formulas

Since Industry 1 consumption is more domestically-oriented than Industry 2 consumption, the weighting of unexpected transfer payments based on domestic production benefits Industry 1, and profit increases are substantially higher than in Table 2 when Industry 1's transfers come exclusively from Industry 1 tariff revenue. When unexpected tariff transfers are weighted based on exports, however, Industry 1 becomes slightly worse off than in Table 2 because consumers in Country 2 import more from Country 1 in Industry 2 than they do in Industry 1, and so the weighting scheme penalizes Industry 1 relative to an industry-only transfer scheme.

[12] If analysis is restricted to profits from the sale of goods in Country 1 and Country 2, the percent change in Country 1 profits with expected payments is smaller with the addition of a third country. The higher profit increase observed in Table 3 is therefore entirely caused by an increase in profit from exports to Country 3.

6 Conclusion

In this paper, I contribute to the partial equilibrium literature by introducing a method for modeling the distribution of tariff revenue as lump-sum transfers to firms. The methodology can be applied to a wide variety of countries or industries, and models both payments that firms do not expect in advance as well as expected payments that firms incorporate into their production and pricing decisions. Firms in the model have heterogenous productivities represented by a Pareto distribution, and this setup allows me to compute equilibrium changes in the fraction of firms that choose to operate in the economy.

I next perform a series of counterfactual simulations in which I estimate the effect of a change in tariffs given hypothetical parameter choices. While the paper does not apply observed data or model any real-life countries, it includes a discussion of what types of data would generally be available and how a researcher might use this data to compute equilibrium solutions to the model. Due to data limitations, the paper presents most results in terms of counterfactual percentage changes rather than counterfactual quantities.

The model is quite versatile and allows its users to perform a multitude of counterfactuals beyond the one presented in Section 5. More broadly, future research in this area can build on Jafarey and Lahiri (2024) and Lahiri and Nasim (2006) by building a partial equilibrium model in which firms receive rebates to compensate them for the tariffs they face on imported intermediate inputs. Such a project would require a multi-stage model, which adds computational complexity to the one-stage model described here. In addition, future researchers could develop a dynamic partial equilibrium model in order to represent industries or situations where the effect of transfer payments is not instantaneous, or alternatively where firms receive transfers at different points in time.

References

[1] Aghion, Philippe, Antonin Bergeaud, Matthieu Lequien, and Marc Melitz. "The Heterogeneous Impact of Market Size on Impacts: Evidence from French Firm-Level Imports." *Review of Economics and Statistics* 106, no. 2 (2022): 608-626.

[2] Ahmad, Saad, and Zeynep Akgul. "Taking Melitz Seriously: A Simple Approach for Identifying Structural Parameters of Trade Models with Firm Heterogeneity." U.S. International Trade Commission Working Paper Series, Working Paper 2018-08-D.

[3] Ahmad, Saad, and Zeynep Akgul. "Using Power Laws to Identify the Structural Parameters of Trade Models with Firm Heterogeneity." U.S. International Trade Commission Working Paper Series, Working Paper 2017-05-B.

[4] Ahmad, Saad, and David Riker. "A Method for Estimating the Elasticity of Substitution and Import Sensitivity by Industry." U.S. International Trade Commission Working Paper Series, Working Paper 2019-05-B.

[5] Alessandria, George, Jiaxiaomei Ding, Shafaat Khan and Carter Mix. "The Tariff Tax Cut: Tariffs as Revenue." NBER Working Paper (2025).

[6] Barbe, Andre, Arthur Chambers, Tamar Khachaturian and David Riker. "Modeling Trade in Services: Multiple Modes, Barriers to Trade, and Data Limitations." U.S. International Trade Commission Working Paper Series, Working Paper 2017-04-B.

[7] Bagwell, Kyle, and Seung Hoon Lee. "Trade Policy under Monopolistic Competition with Firm Selection." *Journal of International Economics* 127 (2020): 103379.

[8] Chaney, Thomas. "Distorted Gravity: Heterogeneous Firms, Market Structure and the Geography of International Trade." *American Economic Review* 98, no. 4 (2008): 1707-1721.

[9] Demidova, Svetlana. "Productivity Improvements and Falling Trade Costs: Boon or Bane?" *International Economic Review* 49, no. 4 (2008): 1437-1462.

[10] Demidova, Svetlana, Andres Rodriguez-Clare. "Trade policy under firm-level heterogeneity in a small economy." *Journal of International Economics* 78 (2009): 100-112.

[11] di Giovanni, Julien, Andrei Levchenko and Romain Rancière. "Power laws in firm size and openness to trade: Measurement and implications." *Journal of International Economics* 85, no. 1 (2011): 42-52.

[12] Eaton, Jonathan, Samuel Kortum and Francis Kramarz. "An Anatomy of International Trade: Evidence from French Firms." *Econometrica* 79, no. 5 (2011): 1453-1498.

[13] Eaton, Jonathan, Samuel Kortum and Francis Kramarz. "Dissecting Trade: Firms, Industries, and Export Destinations." *American Economic Review* 94, no. 2 (2004): 150-154.

[14] Feenstra, Robert, Philip Luck, Maurice Obstfield, and Katheryn Russ. "In Search of the Armington Elasticity." *The Review of Economics and Statistics* 100, vol. 1 (2018): 135-150.

[15] Felbermayr, Gabriel, Benjamin Jung, and Mario Larch. "Optimal Tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model." *Journal of International Economics* no. 89 (2013): 13-25.

[16] Hallren, Ross, and David Riker. "An Introduction to Partial Equilibrium Modeling of Trade Policy." U.S. International Trade Commission Working Paper Series, Working Paper 2017-07-B.

[17] Harrigan, James, Xiangjun Ma and Schlychkov, Victor. "Export Prices of U.S. Firms." *Journal of International Economics* 97, no. 1 (2015): 110-111.

[18] Helpman, Elhanan, Marc Melitz and Stephen Yeaple. "Export Versus FDI with Heterogeneous Firms." *American Economic Review* 94, no. 1 (2004): 300-316.

[19] Jafarey, Saqib and Sajal Lahiri. "Tariff Rebate on Intermediate Inputs for Exportables in a Two-Period Model with Learning-from-Exporting." *International Trade, Resource Mobility and Adjustments in a Changing World* 0 (2024): 21-32.

[20] Keller, Wolfgang, and Stephen Yeaple. "Multinational Enterprises, International Trade, and Productivity Growth: Firm Level Evidence from the United States." Review of Economics and Statistics 91, no. 4 (2019): 821-831.

[21] Khachaturian, Tamar, and David Riker. "A Multi-Mode Partial Equilibrium Model of Trade in Professional Services." U.S. International Trade Commission Working Paper Series, Working Paper 2016-11-A.

[22] Lahiri, Sajal and Anjum Nasim. "Export-promotion under revenue constraints: The case of tariff-rebate on intermediate inputs in Pakistan." *Journal of Asian Economics* 17, no. 2 (2006): 285-293.

[23] Lashkaripour, Ahmad and Volodymyr Lugovskyy. "Profits, Scale Economies, and the Gains from Industrial Policy." *American Economic Review* 113, no. 10 (2023): 2759-2808.

[24] Mayer, Thierry, Marc Melitz, and Gianmarco Ottaviano. "Market Size, Competition, and the Product Mix of Exporters." *American Economic Review* 104, no. 2 (2014): 495-536.

[25] Melitz, Marc. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71, no. 6 (2003): 1695-1725.

[26] Melitz, Marc, and Stephen Redding. "New Trade Models, New Welfare Implications." *American Economic Review* 105, no. 3 (2015): 1105-1146.

[27] Mueller, Patricia, and David Riker. "Practical PE Models of Trade and FDI Policy Uncertainty." U.S. International Trade Commission Working Paper Series, Working Paper 2020-08-A.

[28] Riker, David. "Estimating U.S. Import Penetration into Sub-National Regions." U.S. International Trade Commission Working Paper Series, Working Paper 2019-10-A.

[29] Spearot, Alan. "Unpacking the Long-Run Effects of Tariff Shocks: New Structural Implications from Firm Heterogeneity Models." *American Economic Journal: Microeconomics* 8, no. 2 (2016): 128-167.

7 Appendix: Equilibrium Equations

This section goes in detail through the mathematical processes for using the model to generate an equilibrium.

I begin by deriving expressions for aggregate consumption and prices. Recall that the total value of consumption originating in j and ending in i is

$$\begin{aligned} E_{ji} &= \mu_j \int_{\overline{x}_{ji}}^{\infty} c_{ji}(x) p_{ji}(x) dG_j(x) \\ &= \mu_j \int_{\overline{x}_{ji}}^{\infty} E_i(x) P_i^{\sigma_i - 1} p_{ji}(x)^{1-\sigma_i} dG_j(x) \end{aligned}$$

where the second line comes from replacing $c_{ji}(x)$ with its equivalent given by (1).

Replacing p_{ji} with its equivalent expression from the firms' FOCs and $dG_j(x)$ with the derivative of $G_j(x)$:

$$\begin{aligned} E_{ji} &= \mu_j E_i(x) P_i^{\sigma_i - 1} \int_{\overline{x}_{ji}}^{\infty} \Big(\frac{\tau_{ji}\sigma_i}{(\sigma_i - 1)x}\Big)^{1-\sigma_i} \gamma_j x^{-\gamma_j - 1} dx \\ &= \mu_j E_i(x) P_i^{\sigma_i - 1} \Big(\frac{\tau_{ji}\sigma_i}{(\sigma_i - 1)}\Big)^{1-\sigma_i} \gamma_j \int_{\overline{x}_{ji}}^{\infty} x^{\sigma_i - \gamma_j - 2} dx \\ &= \mu_j E_i(x) P_i^{\sigma_i - 1} \Big(\frac{\tau_{ji}\sigma_i}{(\sigma_i - 1)}\Big)^{1-\sigma_i} \frac{\gamma_j}{\sigma_i - \gamma_j - 1} \big(\lim_{x\to\infty} x^{\sigma_i - 1 - \gamma_j} - \overline{x}_{ji}^{\sigma_i - 1 - \gamma_j}\big) \\ &= \mu_j E_i(x) P_i^{\sigma_i - 1} \Big(\frac{\tau_{ji}\sigma_i}{(\sigma_i - 1)}\Big)^{1-\sigma_i} \frac{\gamma_j \overline{x}_{ji}^{\sigma_i - 1 - \gamma_j}}{\gamma_j - (\sigma_i - 1)} \end{aligned}$$

The last step requires the assumption that $\gamma_j \geq \sigma_i - 1$.

If I observe consumer expenditure share $\beta_{ji} = \frac{E_{ji}}{E_i}$, then I end up with

$$\beta_{ji} = \gamma_j \mu_j P_i^{\sigma_i - 1} \Big(\frac{\tau_{ji}\sigma_i}{(\sigma_i - 1)}\Big)^{1-\sigma_i} \frac{\overline{x}_{ji}^{\sigma_i - 1 - \gamma_j}}{\gamma_j - (\sigma_i - 1)}$$

The evaluation of an aggregate price expression follows a similar process. Recall that the aggregate price in country i is given by

$$P_i^{1-\sigma_i} = \sum_{j=1}^{N} \big[\mu_j \int_{\overline{x}_{ji}}^{\infty} p_{ji}(x)^{1-\sigma_i} dG_j(x)\big]$$

Evaluation of this expression involves the same substitutions for $p_{ji}(x)$ and $dG_j(x)$ that were done in evaluating E_{ji} above. After some algebra and integral calculus we find

$$P_i^{1-\sigma_i} = (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \sum_{j=1}^{N} \mu_j \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} (\tau_{ji})^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - \gamma_j - 1}$$

An equilibrium is characterized by consumption expressions, a price index expression, and finally zero-profit conditions that use the definition of $\overline{x}_{ji}$ as the boundary point where a country j firm is indifferent between selling in country i and shutting down. In other words:

$$\begin{aligned}
f_{ji} &= \pi(\overline{x}_{ji}) \\
&= Y_i P_i^{\sigma_i - 1} p_{ji}(\overline{x}_{ji})^{1-\sigma_i} - \frac{\tau_{ji} Y_i P_i^{\sigma_i - 1} p_{ji}(\overline{x}_{ji})^{-\sigma_i}}{\overline{x}_{ji}} \\
&= Y_i P_i^{\sigma_i - 1} \big((\frac{\sigma_i \tau_{ji}}{(\sigma_i - 1)\overline{x}_{ji}})^{1-\sigma_i} - \frac{\tau_{ji}}{\overline{x}_{ji}} (\frac{\sigma_i \tau_{ji}}{(\sigma_i - 1)\overline{x}_{ji}})^{-\sigma_i} \big) \\
&= Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} \big((\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} - (\frac{\sigma_i}{\sigma_i - 1})^{-\sigma_i} \big) \\
&= Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \frac{1}{\sigma_i}
\end{aligned}$$

The zero-profit condition, consumption equation, and aggregate price equation fully characterize the equilibrium.

7.1 Counterfactual Equilibria

A chief purpose of this model is to investigate counterfactual situations where the tariff rate changes. The standard procedure for this exercise involves using data for an initial equilibrium to calibrate parameters, then solve for a new equilibrium with these parameters where τ_{ji} is different. However, given an inability to observe or calibrate f_{ji} and μ_j, I solve the model in terms of *percent* changes in variables rather than in terms of the variables themselves.

Let's start with fixed costs. In a world where tariffs change, fixed costs are assumed to stay constant. I therefore know that

$$\begin{aligned}
Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \frac{1}{\sigma_i} &= \hat{Y}_i \hat{P}_i^{\sigma_i - 1} \hat{\tau}_{ji}^{1-\sigma_i} \hat{\overline{x}}_{ji}^{\sigma_i - 1} (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \frac{1}{\sigma_i} \\
\Rightarrow P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} &= \hat{P}_i^{\sigma_i - 1} \hat{\tau}_{ji}^{1-\sigma_i} \hat{\overline{x}}_{ji}^{\sigma_i - 1} \\
\Rightarrow \frac{P_i \overline{x}_{ji}}{\tau_{ji}} &= \frac{\hat{P}_i \hat{\overline{x}}_{ji}}{\hat{\tau}_{ji}}
\end{aligned}$$

Rearranging the price equation yields

$$1 = (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \sum_{j=1}^{N} \mu_j \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} (\tau_{ji})^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - \gamma_j - 1} P_i^{\sigma_i - 1}$$

$$1 = (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \sum_{j=1}^{N} \mu_j \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} (\hat{\tau}_{ji})^{1-\sigma_i} \hat{\overline{x}}_{ji}^{\sigma_i - \gamma_j - 1} \hat{P}_i^{\sigma_i - 1} \text{ in counterfactual}$$

The goal is to use my rearranged zero-profit condition to replace $\overline{x}_{ji}$ and P_i, since I do not know either of these two variables. Using algebra I can do some rewriting.

$$\begin{aligned} \hat{\overline{x}}_{ji}^{\sigma_i - 1 - \gamma_j} \hat{P}_i^{\sigma_i - 1} \hat{\tau}_{ji}^{1-\sigma_i} &= (\frac{\hat{\overline{x}}_{ji} \hat{P}_i}{\hat{\tau}_{ji}})^{\sigma_i - 1 - \gamma_j} \hat{P}_i^{\gamma_j} \hat{\tau}_{ji}^{-\gamma_j} \\ &= (\frac{\overline{x}_{ji} P_i}{\tau_{ji}})^{\sigma_i - 1 - \gamma_j} \hat{P}_i^{\gamma_j} \hat{\tau}_{ji}^{-\gamma_j} \end{aligned}$$

Next I turn to the consumption equation. The goal is, again, to isolate x_{ji} and P_i so that I may eventually replace them. Rearranging the items in the consumption equation gives

$$\begin{aligned} \overline{x}_{ji}^{\sigma_i - 1 - \gamma_j} P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} &= \frac{\beta_{ji}(\gamma_j - (\sigma_i - 1))}{\gamma_j \mu_j} (\frac{\sigma_i}{\sigma_i - 1})^{\sigma_i - 1} \\ \Rightarrow (\frac{\overline{x}_{ji} P_i}{\tau_{ji}})^{\sigma_i - 1 - \gamma_j} P_i^{\gamma_j} \tau_{ji}^{-\gamma_j} &= \frac{\beta_{ji}(\gamma_j - (\sigma_i - 1))}{\gamma_j \mu_j} (\frac{\sigma_i}{\sigma_i - 1})^{\sigma_i - 1} \end{aligned}$$

Going back to the price index equation,

$$\begin{aligned} 1 &= (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \sum_{j=1}^{N} \mu_j \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} (\frac{\overline{x}_{ji} P_i}{\tau_{ji}})^{\sigma_i - 1 - \gamma_j} \hat{P}_i^{\gamma_j} \hat{\tau}_{ji}^{-\gamma_j} \\ &= (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i} \sum_{j=1}^{N} \mu_j \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} \frac{\beta_{ji}(\gamma_j - (\sigma_i - 1))}{\gamma_j \mu_j} (\frac{\sigma_i}{\sigma_i - 1})^{\sigma_i - 1} P_i^{-\gamma_j} \tau_{ji}^{\gamma_j} \hat{P}_i^{\gamma_j} \hat{\tau}_{ji}^{-\gamma_j} \\ &= \sum_{j=1}^{N} \beta_{ji} (\frac{\hat{P}_i}{P_i})^{\gamma_j} (\frac{\hat{\tau}_{ji}}{\tau_{ji}})^{-\gamma_j} \end{aligned}$$

I can therefore use algebraic substitutions to rewrite the price index equation in terms of ratios between old and new prices and tariffs, or alternatively the percentage changes in those items. The only unknown object is the ratio $\frac{\hat{P}_i}{P_i}$, and I can solve for this ratio algebraically.

Ratios of equilibrium objects similarly characterize changes in consumption and changes in the cutoff productivity:

$$\frac{\hat{E}_{ji}}{E_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})^{1-\sigma_i}(\frac{\hat{P}_i}{P_i})^{\sigma_i-1}(\frac{\hat{P}_i}{P_i})^{\sigma_i-1-\gamma_j}$$
$$\frac{\hat{\bar{x}}_{ji}}{\bar{x}_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})(\frac{\hat{P}_i}{P_i})^{-1}$$

7.2 Solving for an Equilibrium with Tariff Transfer Payments

When transfer payments enter the picture, the situation in subsection 7.1 above changes slightly. The payments effectively alter the fixed costs for firms to enter the market, so (5) is no longer true. Instead we have that

$$\frac{\hat{\bar{x}}_{ji}}{\bar{x}_{ji}} = (\frac{\hat{\tau}_{ji}}{\tau_{ji}})(\frac{\hat{P}_i}{P_i})^{-1}(\frac{\hat{f}_{ji}}{f_{ji}})^{\frac{1}{\sigma_i-1}}$$
$$\Rightarrow \frac{\hat{\bar{x}}_{ji}\hat{P}_{ji}}{\hat{\tau}_{ji}} = \frac{\bar{x}_{ji}P_{ji}}{\tau_{ji}}(\frac{\hat{f}_{ji}}{f_{ji}})^{\frac{1}{\sigma_i-1}}$$

As before, we want to replace x_{ji} and P_i, but the process is slightly different now that it involves fixed costs.

$$\hat{\bar{x}}_{ji}^{\sigma_i-1-\gamma_j}\hat{P}_i^{\sigma_i-1}\hat{\tau}_{ji}^{1-\sigma_i} = (\frac{\hat{\bar{x}}_{ji}\hat{P}_i}{\hat{\tau}_{ji}})^{\sigma_i-1-\gamma_j}\hat{P}_i^{\gamma_j}\hat{\tau}_{ji}^{-\gamma_j}$$
$$= (\frac{\bar{x}_{ji}P_i}{\tau_{ji}}(\frac{\hat{f}_{ji}}{f_{ji}})^{\frac{1}{\sigma_i-1}})^{\sigma_i-1-\gamma_j}\hat{P}_i^{\gamma_j}\hat{\tau}_{ji}^{-\gamma_j}$$
$$\Rightarrow (\frac{\bar{x}_{ji}P_i}{\tau_{ji}})^{\sigma_i-1-\gamma_j}(\frac{\hat{f}_{ji}}{f_{ji}})^{1-\frac{\gamma_j}{\sigma_i-1}}\hat{P}_i^{\gamma_j}\hat{\tau}_{ji}^{-\gamma_j} = \frac{\beta_{ji}(\gamma_j-(\sigma_i-1))}{\gamma_j\mu_j}(\frac{\sigma_i}{\sigma_i-1})^{\sigma_i-1}$$

So the only difference in the price index equation is the presence of the fixed cost term. Adding the fixed cost term to the price index equation produces (10).

Since the change in fixed costs comes from the addition of lump-sum transfers, $\frac{\hat{f}_{ji}}{f_{ji}}$ is simply $\frac{f_{ji}-r_j}{f_{ji}}$ or $1-\frac{r_j}{f_{ji}}$. I now outline a series of algebraic steps allowing me to rewrite the ratio $\frac{r_j}{f_{ji}}$ given ϕ_{ji}, β_{ji} and Y_i.

I first need to calculate a value for F_{ji}, the total fixed cost payment across all firms shipping from j to i before transfer payments apply. The per-firm fixed cost amount f_{ji} is difficult to calculate, but F_{ji} comes out of equations (2) and (4). Recalling that $m_{ji} = \mu_j\bar{x}_{ji}^{-\gamma_j}$ and $\beta_{ji} = \frac{E_{ji}}{E_j}$, (2) is equivalent to

$$\beta_{ji} = m_{ji}P_i^{\sigma_i-1}(\frac{\tau_{ji}\sigma_i}{(\sigma_i-1)})^{1-\sigma_i}\frac{\bar{x}_{ji}^{\sigma_i-1}}{\gamma_j-(\sigma_i-1)}$$

Furthermore, multiplying both sides of (4) by m_{ji} changes it to

$$m_{ji}f_{ji} = F_{ji} = Y_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} (\frac{\sigma_i}{\sigma_i - 1})^{1-\sigma_i}$$

This equation is almost exactly the same as the rewritten version of (2). Putting the two equations together I get[13]

$$F_{ji} = \beta_{ji} Y_i \frac{\gamma_j - (\sigma_i - 1)}{\gamma_j \sigma_i}$$

Now that I have derived a value for F_{ji} in terms of observed quantities, let's use it. Multiplying both numerator and denominator by m_{ji}, the baseline firm participation level, and replacing r_j with R_j gives

$$\begin{aligned} \frac{R_j \frac{m_{ji}}{\sum_k \hat{m}_{jk}}}{f_{ji} m_{ji}} &= \frac{R_j}{F_{ji}} (\frac{\sum_k \hat{m}_{jk}}{m_{ji}})^{-1} \\ &= \frac{R_j}{F_{ji}} (\frac{\sum_k m_{jk} \frac{\hat{m}_{jk}}{m_{jk}}}{m_{ji}})^{-1} \end{aligned}$$

Given information on the fraction of firms that export to each destination, I know $\phi_{jk} = \frac{m_{jk}}{m_{jj}}$ for all k and hence $\frac{\phi_{jk}}{\phi_{ji}} = \frac{m_{jk}}{m_{ji}}$ for all k and i. Putting this into the above equation gives

$$\frac{R_j}{F_{ji}} (\frac{\sum_k \left(m_{ji} \frac{\phi_{jk}}{\phi_{ji}}\right) \left(\frac{\hat{m}_{jk}}{m_{jk}}\right)}{m_{ji}})^{-1} = \frac{R_j}{F_{ji}} (\sum_k (\frac{\phi_{jk}}{\phi_{ji}})(\frac{\hat{m}_{jk}}{m_{jk}}))^{-1}$$

Remember that the ratio $\frac{\hat{m}_{jk}}{m_{jk}}$ is equal to $\left(\frac{\hat{x}_{jk}}{x_{jk}}\right)^{-\gamma_j}$. Once that substitution is made, the ratio $\frac{r_j}{F_{ji}}$ can be written in terms of known quantities and counterfactual ratios.

Finally, let's derive the formula (11) that governs total change in firm participation. Remember that ϕ_{ji}, the fraction of domestically producing firms in j that also ship to i, is equivalent to $(\frac{\overline{x}_{ji}}{\overline{x}_{ii}})^{-\gamma_j}$. For any two destination countries i and n, we therefore know that

$$\frac{\phi_{ji}}{\phi_{jn}} = (\frac{\overline{x}_{ji}}{\overline{x}_{ii}})^{-\gamma_j} (\frac{\overline{x}_{ii}}{\overline{x}_{jn}})^{-\gamma_j} = (\frac{\overline{x}_{ji}}{\overline{x}_{jn}})^{-\gamma_j} \tag{12}$$

[13] Note that operating profit, or profit before fixed costs, along route ji is given by $\frac{1}{\sigma_i}\beta_{ji}Y_i$. Since $\gamma_j - (\sigma_i - 1) < \gamma_j$, operating profit along route ji will always be higher than F_{ji}.

I can then rewrite base-period firm participation m_j as follows:

$$\begin{aligned} m_j &= \mu_j \sum_{i=1}^{N} \overline{x}_{ji}^{-\gamma_j} \\ &= \mu_j \sum_{i=1}^{N} (\frac{\phi_{ji}}{\phi_{jn}}) \overline{x}_{jn}^{-\gamma_j} \\ &= \frac{\mu_j}{\phi_{jn}} \sum_{i=1}^{N} \phi_{ji} \overline{x}_{jn}^{-\gamma_j} \end{aligned}$$

I rewrite $\hat{m}_j$ using a similar process, but with one extra step because the shares ϕ_{ji} are only known in the baseline. I first rewrite $\hat{m}_j$ in terms of counterfactual changes:

$$\hat{m}_j = \mu_j \sum_{i=1}^{N} \hat{\overline{x}}_{ji}^{-\gamma_j} = \mu_j \sum_{i=1}^{N} \overline{x}_{ji}^{\gamma_j} (\frac{\hat{\overline{x}}_{ji}}{\overline{x}_{ji}})^{-\gamma_j}$$

and then I can use (12) to substitute for $\overline{x}_{ji}$.

$$\begin{aligned} \hat{m}_j &= \mu_j \sum_{i=1}^{N} (\frac{\phi_{ji}}{\phi_{jn}}) \overline{x}_{jn}^{-\gamma_j} (\frac{\hat{\overline{x}}_{ji}}{\overline{x}_{ji}})^{-\gamma_j} \\ &= \frac{\mu_j \overline{x}_{jn}^{-\gamma_j}}{\phi_{jn}} \sum_{i=1}^{N} \phi_{ji} (\frac{\hat{\overline{x}}_{ji}}{\overline{x}_{ji}})^{-\gamma_j} \end{aligned}$$

In comparing $\hat{m}_j$ and m_j, multiple terms cancel out, including μ_j and any quantity indexed by n. Dividing $\hat{m}_j$ by m_j therefore gives me (11), which is in terms of counterfactual changes and observed inputs.