Modeling Transfers of Tariff Revenue to Firms within a Multi-Stage Partial Equilibrium Model

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Abstract

In this paper, I set up and solve a partial equilibrium model that incorporates two stages of production and the distribution of tariff revenue to firms. Imports of intermediate and final goods both face tariffs, and tariff rates at each stage of production may differ from one another. Intermediate input producers are modeled as perfectly competitive, but final good firms have a continuum of heterogeneous productivities: final goods firms then enter or exit the market depending on whether their productivity allows them to make a profit higher than their fixed costs of operating. Theoretical simulations illustrate that increases in intermediate goods tariffs and increases in final goods tariffs have quite different counterfactual effects, but the presence of two production stages does not meaningfully change results in a counterfactual where only final goods tariffs change. I conclude my discussion by explaining how the model setup can accommodate alternative modeling choices, such as different market structures for intermediate producers or different production functions for final goods producers.

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1 Introduction

Countries have multiple potential goals when they levy tariffs, including the raising of revenue, supporting of domestic production, and protection of national security. However, tariffs on imports of intermediate goods can also raise costs for domestic firms that use these goods as inputs in their production. The distribution of tariff revenue to such domestic firms could compensate them for trade disruptions, and potentially keep some firms from going out of business. For example, the U.S. Tariff Act of 1789 established duty drawbacks, which generally refund the duties paid on imports if those imports are used to manufacture another project that is then exported. As documented in Lahiri and Nasim (2006), Pakistan also provided refunds to exporters for duties on their imported inputs, in order to increase Pakistan's export competitiveness relative to that of other nations. In this paper, I set up a static partial equilibrium model that incorporates tariff transfer payments to firms who use imported intermediates as inputs in production.

The model described in this paper is able to represent any number of countries or industries, as well as any user's choice of country or industry. Domestic firms in the model use both domestically produced goods and imported goods as an input in their production process, and both imports of production inputs and imports of final goods are subject to tariffs. The government can then distribute lump-sum payments to these firms based on the revenue collected from these tariffs. The modeling framework accommodates different methods for calculating the payments; for example, transfer payments could benefit all firms regardless of their exposure to imported inputs, or they could prioritize the firms with the greatest level of exposure to imported inputs.

Similar to Phillips (2025), this model follows Melitz (2003) by featuring heterogeneous final good-producing firms with varying productivities. Each origin-destination pair ji, at both stages of production, has a cutoff productivity level below which firms in country j cannot make a profit by exporting to i. The model augments the structure originally proposed by Phillips (2025) by including two stages of production, with perfectly competitive firms producing intermediate inputs that the final good firms then purchase. Lump-sum transfer payments effectively reduce the minimum productivity necessary for final good firms to stay in business.

I discuss how to compute equilibria in counterfactual situations where tariffs increase. Due to the difficulty of calibrating certain model parameters, this computational method solves for the percentage changes between baseline and counterfactual equilibria, rather than the absolute values of counterfactual equilibria. Simulation results indicate that variations in tariff rates and input expenditure shares have a large effect on counterfactual changes in firm participation and profits, with the price elasticity of input supply playing a lesser role. Tariffs on final goods and tariffs on intermediate goods play quite different roles in driving modeling results, as the former predict increases in domestic production while the latter can increase production costs and thereby lower domestic production. However, the addition of an intermediate stage of production to the model does not substantially change results in a counterfactual where only final goods tariffs increase.

The rest of the paper proceeds as follows. Section 2 introduces a two-stage partial equilibrium model with heterogeneous firm productivities, including a lump-sum distribution of tariff revenue. Section 3 goes through the model equilibrium and solution method, and Section 4 presents the results of solving the model using hypothetical parameters. Section 5 discusses several ways that users can adjust the model to represent different industries or economic environments. Section 6 concludes.

1.1 Literature Review

Classic models of international trade most frequently model transfers of tariff revenue as a lump-sum payment to *consumers*, rather than firms, and these transfers are typically one of many model features rather than the main focus of analysis. Phillips (2025) and Alessandria et al. (2025) discuss transfer payments for tariffs levied on final goods, while Jafarey and Lahiri (2024) and Lahiri and Nasim (2006) explicitly focus on optimal rebates for tariffs paid on intermediate inputs. The latter two papers discuss intermediate inputs for Pakistan specifically, and their methodology focuses on calculating optimal rebate levels rather than modeling equilibrium outcomes for a wide selection of rebate structures and values.

The literature on intermediate goods tariffs generally, without specific reference to tariff-related payments, is much broader. Papers involving theoretical modeling or structural estimation have frequently incorporated multiple stages of production into their models in order to contrast the effects of intermediate tariffs with the effects of final-stage tariffs. Antràs et al. (2024), Caliendo et al. (2023), and Lashkaripour and Lugovskyy (2023) all build structural models in order to solve for optimal tariffs at different stages of production. Meanwhile, Grossman, Helpman and Redding (2024) and Lashkaripour (2021) build similar models in order to estimate the welfare effects of various protectionist policies, while Kohn, Leibovici and Szkup (2023) estimates how the effects of lower intermediate goods tariffs depend on a country's level of financial development.

This paper also contributes to a series of working papers by Commission staff that incorporate the heterogeneous-productivity modeling structure first proposed by Melitz (2003) into a standard partial equilibrium framework. Barbe, Chambers, Khachaturian, and Riker (2017) and Mueller and Riker (2020) discuss this embedding process more generally, and Khachaturian and Riker (2016) uses a heterogeneous-productivity partial equilibrium model to analyze the effect of decreased fixed costs on international trade in services. The heterogeneous-productivity partial equilibrium model in Phillips (2025) most closely resembles this one, but the addition of intermediate inputs into the model adds substantial new dimensions of analysis.

2 Model

Because this model is a partial equilibrium model, industries have separate equilibrium conditions from one another, and so the equations in this section do not index variables by industry.

2.1 Consumers

Consumers in country i consume final goods from any one of N origin countries, indexed by j. The model further subdivides consumption from each origin according to varieties z, and these consumers choose among origins and varieties to maximize the following CES utility function:

$$\left(\sum_{j=1}^{N} \int_{0}^{m_{ji}} c_{ji}(z)^{\frac{\sigma_i - 1}{\sigma_i}} dz\right)^{\frac{\sigma_i}{\sigma_i - 1}}$$

where σ_i is the elasticity of substitution and m_{ji} is an equilibrium allocation representing the total measure of firms that export from j to i.

Let E_i denote total consumer expenditure on a given industry in country i. As is common in partial equilibrium models,[1] I assume that the total elasticity of demand is -1, so E_i remains constant in all counterfactual situations. The consumer's first–order conditions lead to the following equation:

$$E_{ji}(z) = E_i P_i^{\sigma_i - 1} P_{ji}(z)^{1-\sigma_i} \ \forall z \forall i \forall j \tag{1}$$

where P_i is the aggregate price index for country i and $P_{ji}(z)$ is the price of variety z when shipped from country j to country i. The aggregate price index P_i is given by

$$P_i^{1-\sigma_i} = \sum_{j=1}^{N} \int_{0}^{m_{ji}} P_{ji}(z)^{1-\sigma_i} dz$$

2.2 Firms

Since consumers only consume final goods, the consumers' side is the same in the model as it would be in a model without intermediate goods. The firms' component of the model is different, however, because final goods firms produce their goods by aggregating inputs from various sources.

A continuum of firms, each with productivity x, produce final goods. I can assume, without loss of

[1] See Riker and Hallren (2017).

generality, that each productivity x maps to variety z as featured in the consumers' utility function. A firm in country j seeking to export to country i chooses its price to maximize the following profit function:

$$P_{ji}(x)y_{ji}(x) - \frac{P_{ju}\tau_{ji}y_{ji}(x)}{x} - f_{ji}$$

where f_{ji} refers to the fixed cost of exporting from j to i. First-order conditions lead to the formula

$$P_{ji}(x) = \frac{\sigma_i \tau_{ji}}{(\sigma_i - 1)x}$$

Final good firms pay a unit production cost P_{ju} that aggregates the prices of all intermediate inputs.[2] The u subscript in general denotes equilibrium prices and allocations related to the intermediate stage of production.

Final production in country j is a CES aggregate of intermediate inputs with elasticity of substitution λ_j:

$$y_{ju}(x)^{\frac{\lambda_j - 1}{\lambda_j}} = \sum_{i=1}^{N} \big(\frac{y_{iju}}{\tau_{iju}}\big)^{\frac{\lambda_j - 1}{\lambda_j}}$$

τ_{iju}, the tariff charged by country i on intermediate production from i, is the crucial mechanism in this model. If tariffs increase across both stages of production, then country i firms using intermediate inputs have to pay a higher price for those inputs, an effect that works in opposition to the benefits country i firms get from increased tariffs on imports of final goods. First-order conditions determine that

$$y_{iju} = y_{ju} P_{ju}^{\lambda_j} P_{iju}^{-\lambda_j} \tag{2}$$

$$P_{ju}^{1-\lambda_j} = \sum_{i=1}^{N} (P_{iju}\tau_{iju})^{1-\lambda_j} \tag{3}$$

Intermediate goods are produced in a perfectly competitive environment given by

$$y_{iju} = P_{iju}^{\varepsilon_{ij}} \tag{4}$$

where ε_{ij} is the price elasticity of supply for intermediate goods exported from i to j. Model users can simplify calibration and equilibrium equations by assuming $\varepsilon_{ij} = \varepsilon \ \forall i \forall j$, but this assumption would also lead to a loss of heterogeneity in countries' ability to source domestically produced inputs.

[2] This price differs from the single-stage model in Phillips (2025). The single-stage model simply features a unit cost of producing the good, and I normalize this unit cost to 1.

2.3 Productivity Distribution

Productivity x follows a Pareto distribution, with distribution $F_i(x)$:

$$F_i(x) = 1 - x^{-\gamma_i}$$

The properties of the Pareto distribution ensure the existence in equilibrium of a cutoff productivity $\bar{x}_{ji}$ for exports from j to i, such that final goods producers in j with productivities lower than $\bar{x}_{ji}$ do not export to i. Increased production costs for final goods producers mean that a greater number of them are unable to recoup their fixed costs of production, and $\bar{x}_{ji}$ goes up.

I use μ_j to represent the measure of *potential* firms in country j. Properties of the Pareto distribution tell us that, in equilibrium, $m_{ji} = \mu_j \bar{x}_{ji}^{-\gamma_j}$.

2.4 Equilibrium Equations

Equations (2)-(4) come from the first stage of production. Equilibrium equations for final goods expenditure and prices are identical to those in Phillips (2025), with the only difference being that the input price aggregate P_{ju} replaces unit production costs. These equations are

$$E_{ji} = \mu_j E_i P_i^{\sigma_i - 1} \left(\frac{\tau_{ji} P_{ju} \sigma_i}{(\sigma_i - 1)}\right)^{1-\sigma_i} \frac{\overline{x}_{ji}^{\sigma_i - 1 - \gamma_j}}{\gamma_j - (\sigma_i - 1)} \tag{5}$$

$$P_i^{1-\sigma_i} = \frac{\gamma_j}{\gamma_j - (\sigma_i - 1)} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \sum_{j=1}^{N} \mu_j (\tau_{ji})^{1-\sigma_i} P_{ju}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - \gamma_j - 1} \tag{6}$$

$$f_{ji} = E_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} P_{ju}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \frac{1}{\sigma_i} \tag{7}$$

Market clearing dictates that the total quantity of aggregated intermediate inputs in j must equal the total quantity of final goods produced in j, which is equivalently the total of j's exports to all destinations. Furthermore, the supply of intermediate inputs exported from i to j must equal j's demand for inputs produced in i, combining (2) and (4). In numerical terms:

$$y_{ju} = \sum_{i=1}^{N} y_{ji} \tag{8}$$

$$P_{iju}^{\varepsilon_{ij}} = y_{iju} = y_{ju} P_{ju}^{\lambda_j} (P_{jiu} \tau_{jiu})^{-\lambda_j} \tag{9}$$

2.5 Transfer Payments

Let R_i represent the total revenue transferred to firms across the entire economy of country i, and r_i be the transfer payment received by each firm within country i. I assume that final goods firms are the exclusive recipients of the transfer payments, as domestic producers of intermediate inputs would most likely benefit from the introduction of tariffs on those inputs. Assuming that all firms within a country receive the same transfer amount, the following identity relates R_i and r_i:

$$r_i = \frac{R_i}{\mu_i \sum_{j=1}^{N} \bar{x}_{ij}^{-\gamma_j}}$$

Total tariff revenue across the entire economy in country i comes from two sources: revenue from tariffs on intermediate goods and revenue from tariffs on final goods. The total revenue value is

$$\sum_{j \neq i}^{N} \left[(\tau_{ji} - 1) \int_{\overline{x}_{ji}}^{\infty} E_{ji}(x) dF_j(x) + (\tau_{jiu} - 1) E_{jiu} \right]$$

and total transfer payments are given by

$$R_i = \psi_i \sum_{j \neq i}^{N} \left[(\tau_{ji} - 1) \int_{\overline{x}_{ji}}^{\infty} E_{ji}(x) dF_j(x) + (\tau_{jiu} - 1) E_{jiu} \right]$$

The ψ_i parameter ranges from 0 to 1, allowing modelers to vary the magnitude of the payment rather than limit themselves to situations where firms either receive no transfer payments or receive payments equal to the entirety of the collected tariff revenue.

In a given simulation, transfers may be expected or unexpected, which determines whether firms consider these transfer payments when making production decisions. The equilibrium in the latter situation would be the solution to a fixed-point problem where R_i is equal to some function of equilibrium imports and industry-specific tariff rates, on both intermediate and final goods. In the case of anticipated transfer payments, equation (7) becomes

$$f_{ji} - r_j = E_i P_i^{\sigma_i - 1} \tau_{ji}^{1-\sigma_i} P_{ju}^{1-\sigma_i} \overline{x}_{ji}^{\sigma_i - 1} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{1-\sigma_i} \frac{1}{\sigma_i}$$

with transfers effectively decreasing the fixed costs of production. All other equations remain unchanged.

Transfer payments could also differ in terms of how they are distributed. Payments could be industry-specific, meaning that firms within each industry receive the total value of revenue collected from imports in that industry, or each industry could receive a share of tariff revenue collected across the entire economy.

Furthermore, the distribution of tariff revenue to firms could disproportionately benefit firms with higher use of imported inputs. Because simulations in Phillips (2025) already explored different transfer schemes in some detail, this paper will not do so, and interested readers are encouraged to refer to that earlier paper.

3 Calibration and Estimation

3.1 Calibration Strategy

The list of parameters that are both necessary and sufficient to solve for an equilibrium includes γ, σ, λ, τ, τ_u, f, μ, and ε. Broadly speaking, information on intermediate inputs comes from two major sources: lists that distinguish products by the role they play in the production process; and matrices of input-output data. The World Integrated Trade Solution (WITS) provides an example of the former, namely a list of Harmonized System 6-digit (HS6) products classified by the stage of production processing at which those products are most commonly used. In the event that researchers are modeling a more aggregated industry or an entire economy, they may simply break down the industry or economy into its HS6 components, then aggregate the relevant parameters separately for intermediate and final goods. Supply and use tables, provided for the United States by the BEA and for OECD countries by the OECD, provide the proportion of production in a given industry that is consumed as final goods and used in the production process for other industries.[3]

Data on tariffs τ_{ji} may be found using the Harmonized Tariff System (HTS) at the USITC's DataWeb. WITS categorizations of HS products by end use enable users to compute tariffs specifically for intermediate products and for final goods products.

Two possible methods for estimating elasticities of substitution σ and λ are the markup method, introduced in Ahmad and Riker (2019), and the trade cost method, as discussed in Riker (2019). As shown in Phillips (2024), model users can obtain elasticity estimates specifically for intermediate or final products by classifying HTS product codes as 'intermediate' or 'final' and applying the trade cost method to those specific classifications. The markup method works less well here, because the data used to apply it are only available at the NAICS 4-digit level, and many NAICS 4-digit industries contain both intermediate and final products.

Methods for estimating γ can likewise distinguish between intermediate and final stages of production. The firm-level data used to estimate these parameters may be subdivided into intermediate goods and final goods categories, enabling users to estimate the Pareto shape parameter separately for the two categories. In the event that firm industry classifications are insufficiently granular, one may simply use the same shape

[3] The World Input-Output tables are another source of input-output matrices, but have not been updated since 2016.

parameter for both countries or industries without significantly affecting results.

Soderbery (2018) provides export supply elasticities for pairs of countries at the HS 4-digit level. Soderbery (2015) also provides import supply elasticities at the HTS 8- and HTS 10-digit levels for the United States only. The use of the HS 4-digit elasticity estimates from Soderbery (2018) presents challenges for computing input or output-specific elasticities, as WITS and other sources categorize by use based on the HS 6-digit level. If using parameters from this paper, the researcher could assign to HS 6-digit products the elasticities estimated for the HS 4-digit industries to which they belong, without substantially affecting model outputs.

Fixed costs f and firm measures μ are also necessary to solve for an equilibrium, but these parameters do not have any clear counterpart in the data, nor are there typical values for these parameters that I could impose in theoretical simulations such as the ones performed in this paper. I therefore discuss how to use alternative model outputs that do exist in the data, namely expenditure shares and the share of firms that export to each destination.

I use the symbol β to represent expenditure shares, with $\beta_{ji} = \frac{E_{ji}}{E_i}$ representing an expenditure share for final goods and β_{iju} representing the value of inputs from i as a fraction of total production costs in j. Although expenditure shares can generally be calibrated using available data on imports and imputed measurements of domestic consumption[4], measurements of domestic consumption are not typically available at a sufficiently granular level to distinguish between intermediate and final goods. With imputations of domestic expenditure shares both for inputs and for final production, and more detailed data on imports for final goods product categories and intermediate goods product categories, model users can construct values for β_{ji} and β_{jiu}.

Finally, the symbol ϕ_{ji} denotes the share of domestically producing final goods firms in j that ship to destination i. While this measure typically cannot be computed without using detailed firm-level data, modelers can still input plausible values of the fraction of firms exporting to each destination, and in a two-country model ϕ_{ji}, $j \neq i$ represents the fraction of firms in j that export at all. [5]

3.2 Solving the Model

This subsection discusses how to solve the model for equilibrium changes in a counterfactual with different rates for tariffs on intermediate goods or tariffs on final goods. The modeling framework could also be used to examine a situation where tariffs and fixed costs of production both change.

Due to the absence of information on μ and f, solving for an equilibrium requires rearranging equilib-

[4]The ITC's ITPD Database is one source that provides such measurements.

[5]If the model were to feature lump-sum transfers to intermediate good producers as well as final goods producers, then its users would need to know ϕ_{ji} separately for final goods producers and for intermediate goods producers.

rium equations so that they are in terms of changes in allocations rather than the levels of the allocations themselves. Let $\hat{y}$ be the counterfactual level of a variable y, so that $\hat{\tau}_{ji}$ represents counterfactual tariff levels and τ_{ji} represents baseline tariffs. Likewise, let $\tilde{y}$ represent the fractional counterfactual change in a variable y, so that $\tilde{\tau}_{ji} = \frac{\hat{\tau}_{ji}}{\tau_{ji}}$. I start by explaining how to solve for an equilibrium without any transfer payments, then explain how the addition of transfers into the model changes the solution methodology.

In an equilibrium without transfer payments, fixed costs do not change in the counterfactual, so $\hat{f}_{ji} = f_{ji}$ and $\tilde{f}_{ji} = 1$. A rearrangement of (5) and (7) leads to

$$\tilde{E}_{ji} = \tilde{P}_i^{\sigma_i - 1}\big(\tilde{\tau}_{ji}\tilde{P}_{iu}\big)^{1-\sigma_i}\tilde{\overline{x}}_{ji}^{\sigma_i - 1 - \gamma_j}$$

$$\tilde{P}_{iu}\tilde{\tau}_{ji} = \tilde{P}_i\tilde{\overline{x}}_{ji}$$

and the counterfactual change in firm participation is

$$\tilde{m}_{ji} = \tilde{\overline{x}}_{ji}^{-\gamma_j}$$

As shown in Phillips (2025), the price index equation (6) can be rewritten using (5) and (7) to eliminate $\overline{x}_{ji}$. However, while the input price in Phillips (2025) was normalized to 1, the input price in this paper is a CES aggregate of individual input prices. The price equation is therefore

$$1 = \sum_{j=1}^{N} \beta_{ji}\big(\frac{\hat{P}_i}{P_i}\big)^{\gamma_j}\big(\frac{\hat{P}_{ju}}{P_{ju}}\big)^{-\gamma_j}\big(\frac{\hat{\tau}_{ji}}{\tau_{ji}}\big)^{-\gamma_j} \tag{10}$$

$$= \sum_{j=1}^{N} \beta_{ji}\tilde{P}_i^{\gamma_j}\tilde{P}_{ju}^{-\gamma_j}\tilde{\tau}_{ji}^{-\gamma_j} \tag{11}$$

Each country has one price equation, featuring the unknowns $\tilde{P}_i$ and $\{\tilde{P}_{ju}\}_{j=1}^{N}$. In total, there are N equations, one for each country, and $2N$ unknown prices.

I complete the system by rewriting (3), (8) and (9) in terms of counterfactual changes rather than levels. A trivial rewriting of (9) leads to

$$\tilde{P}_{iju}^{\varepsilon_{ij}} = \tilde{y}_{ju}\tilde{P}_{ju}^{\lambda_j}(\tilde{\tau}_{iju}\tilde{P}_{iju})^{-\lambda_j} \tag{12}$$

and a rewriting of (3) using intermediate expenditure shares β_u leads to

$$\tilde{P}_{ju}^{1-\lambda_i} = \sum_{j=1}^{N} \beta_{iju}(\tilde{P}_{iju}\tilde{\tau}_{iju})^{1-\lambda_j} \tag{13}$$

Note that if I combine the versions of (12) for origin countries i and j, I can eliminate $\tilde{y}_{ju}$ and $\tilde{P}_{ju}$ to get

$$\frac{\tilde{P}_{iju}^{\lambda_j+\varepsilon_{ij}}}{\tilde{P}_{jju}^{\lambda_j+\varepsilon_{jj}}} = \tilde{\tau}_{iju}^{1-\lambda_j} \tag{14}$$

since $\tau_{iiu} = 1$ and $\tilde{\tau}_{iiu} = 1$. I can then replace $\tilde{P}_{jiu},\ j \neq i$ in (11) to get

$$\begin{aligned}\tilde{P}_{ju}^{1-\lambda_j} &= \sum_{j=1}^{N} \beta_{iju} \big(\tilde{P}_{jiu}^{\frac{\varepsilon_{jj}+\lambda_j}{\varepsilon_{ij}+\lambda_j}} \tilde{\tau}_{jiu}^{\frac{-\lambda_j}{\varepsilon_{ij}+\lambda_j}} \tilde{\tau}_{jiu}\big)^{1-\lambda_j} \\ &= \sum_{j=1}^{N} \beta_{iju} \big(\tilde{P}_{jiu}^{\frac{\varepsilon_{jj}+\lambda_j}{\varepsilon_{ij}+\lambda_j}} \tilde{\tau}_{jiu}^{\frac{\varepsilon_{ij}}{\varepsilon_{ij}+\lambda_j}}\big)^{1-\lambda_j}\end{aligned} \tag{15}$$

which contains two unknown price ratios: $\tilde{P}_{ju}$ and $\tilde{P}_{iju}$. To close the system, I rewrite (8). (8) becomes

$$\tilde{p}_{iju}^{\varepsilon_{ij}} = \tilde{P}_{ju}^{\lambda_j} (\tilde{P}_{iju} \tilde{\tau}_{iju})^{-\lambda_j} \frac{1}{\tilde{P}_{ju}} \frac{\sum_{i=1}^{N} \frac{\sigma_i - 1}{\sigma_i} E_i \beta_{ji} \tilde{P}_i^{\gamma_j} \tilde{P}_{ju}^{-\gamma_j}}{\sum_{j=1}^{N} \frac{\sigma_i - 1}{\sigma_i} E_i}. \tag{16}$$

In an equilibrium with transfer payments, fixed costs change in the counterfactual for all firms that receive the payments.

3.2.1 Solving the Model with Transfer Payments

Lump-sum transfer payments to final goods firms in country i effectively decrease the fixed costs necessary for them to operate. With such a change in fixed costs, $\tilde{f}_{ij} \neq 1$ which alters equation (7). Equation (7) now rearranges to become

$$\frac{\hat{\bar{x}}_{ji} \hat{P}_{ji}}{\hat{\tau}_{ji} \hat{P}_{ju}} = \frac{\bar{x}_{ji} P_{ji}}{\tau_{ji} P_{ju}} \big(\frac{\hat{f}_{ji}}{f_{ji}}\big)^{\frac{1}{\sigma_i - 1}}$$

and (11) becomes

$$1 = \sum_{j=1}^{N} \beta_{ji} \tilde{P}_i^{\gamma_j} \tilde{P}_{ju}^{-\gamma_j} \tilde{\tau}_{ji}^{-\gamma_j} \tilde{f}_{ji}^{1 - \frac{\gamma_j}{\sigma_i - 1}}$$

This change in fixed costs does not affect equations that involve intermediate inputs, because only final goods firms receive transfer payments. However, if final goods firms expect in advance to receive the payments, transfers would affect intermediate producers indirectly because final goods firms' production, and thus their demand for inputs, would increase.

4 Simulation Results

In this section, I present results of simulating the model after selecting a series of values for the parameter inputs. These exercises showcase the model's capabilities by demonstrating how changing certain inputs impacts model results.

All simulations feature two countries, which both have firm productivity shape parameter $\gamma = 4$, elasticity of substitution $\sigma = 3$, and input elasticity of substitution $\lambda = 3$. Both countries also have aggregate expenditure $E = 100$, and initially spend 70% of their final expenditure on domestically produced goods compared with 30% on imported goods. Finally, $\phi_{12} = .2$, meaning that 20 percent of domestically producing Country 1 firms also export to Country 2. All results that I present are for Country 1[6], and in all simulations involving transfer payments only Country 1 firms receive the payments.

I begin with simulations in which I vary either the intermediate input tariff or the final goods tariff. I then hold tariff policy fixed and explore the effects of modifying other input-relevant parameters, namely input expenditure shares β_u and price elasticities of input supply ε.

4.1 Tariff Simulations

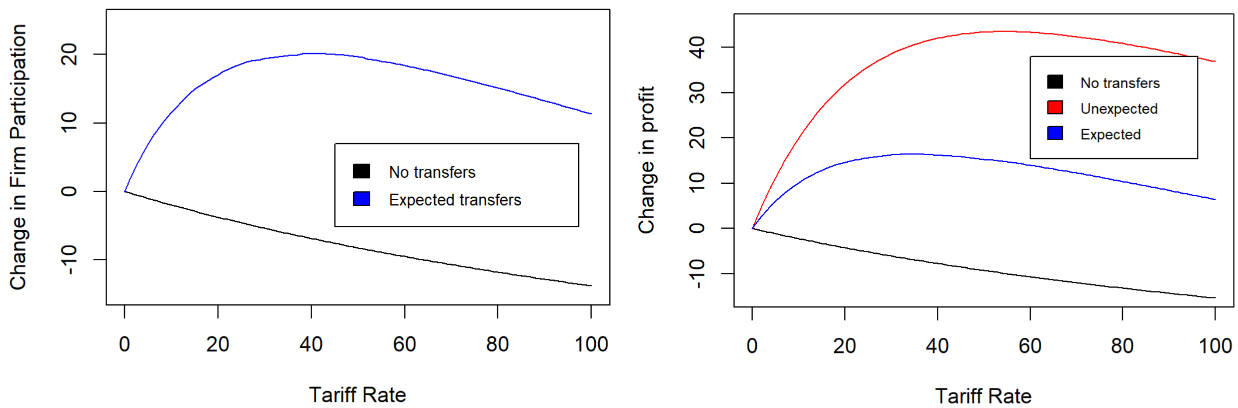
The simulations in this section focus on final goods tariffs τ and intermediate goods tariffs τ_u, and consist of five counterfactual scenarios. The first two scenarios increase tariffs on intermediate goods, first unilaterally and then reciprocally, without any change in tariffs on final goods. The third and fourth scenarios increase tariffs on final goods, first unilaterally and then reciprocally, without any change in tariffs on intermediate goods. The fifth scenario increases tariffs on both intermediate goods and final goods for both countries.

Both countries have price elasticity of supply $\varepsilon = 1.1$, and in the initial 'baseline' equilibrium producers in both countries initially spend 70% of their input expenditure on imported goods. Furthermore, final goods firms in country 1 receive 50% of tariff revenue as transfer payments ($\psi = 0.5$) while as noted above Country 2 firms do not receive transfers. This choice of parameters applies to all five simulations.

In Figure 1, I vary Country 1's increase in tariffs on intermediate goods from 0 to 100%, so that $\tilde{\tau}_{21u}$ ranges from 1 to 2. Figure 1a) displays the percent change in total final good firm participation, $\tilde{m}_1$ or $\frac{\hat{m}_1}{m_1} - 1$, and Figure 1b) displays the percent change in total country 1 profits, $\tilde{\pi}_1$ or $\frac{\hat{\pi}_1}{\pi_1} - 1$. Country 2's intermediate tariffs and both countries' final goods tariffs do not change in the counterfactual.

Without transfer payments, both firm participation and profit changes are negative and monotonically decreasing in the tariff change (Figure 1). Higher intermediate goods tariffs result in costlier inputs, which both lowers the profits that final goods firms can make and limits the fraction of final goods firms able to

[6] Because the results that I present focus on Country 1, I do not need to set a value for ϕ_{21}; providing ϕ values for Country 2 is not necessary to compute counterfactual changes in Country 1.



(a) Percent change in firm participation (b) Percent change in total profit

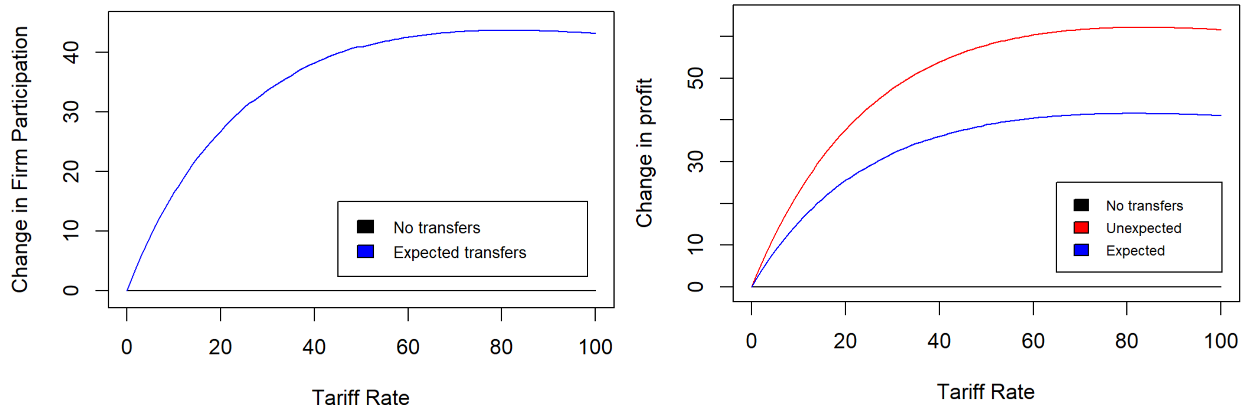
Figure 1: Counterfactual outcomes as a function of intermediate tariff increases in Country 1 only

recoup their fixed costs. With transfers, however, firm participation and profits are higher compared to a scenario where Country 1 does not increase its intermediate goods tariffs. The expectation of transfer payments effectively lowers final goods firms' fixed cost of production, counteracting higher input costs and resulting in substantially higher firm participation (Figure 1a). The increase in profits from unexpected transfers is higher than the increase in profits from expected transfers because expected transfers result in an influx of less productive firms that undercuts market power for the firms already producing.

Figure 1 also shows that counterfactual increases in firm participation and profit increase rise at first, but then fall. As tariff rates increase, the disadvantage of paying higher input costs begins to exceed the advantage of receiving transfers. The level of tariff revenue collected, and the transfer of this tariff revenue to firms, is nevertheless high enough that counterfactual changes in Figure 1a) and Figure 1b) continue to be positive even when intermediate tariffs are 100%.

I next model reciprocal tariffs by increasing intermediate goods tariffs for *both* countries by the same rate. As shown in Figure 2a, firm participation and profits do not change in an equilibrium without transfers. If both countries are identical and raise their tariffs by identical amounts, then in any counterfactual without transfers aggregate input prices P_{iu} and P_{ju} will increase by the same percentage. (11) implies that this increase passes through completely into $\tilde{P}_i$, the change in country i's aggregate price of final goods, so that $\tilde{P}_i$ increases by the same percentage as well. With no change in final goods tariffs and no change in the final goods price relative to the input price, the value of firms' sales will not change and they have no incentive to enter or leave the market.

If transfers to Country 1 firms are present, however, increases in firm participation and total profit are higher than those in Figure 1, and the increases rise monotonically rather than rising at first and then

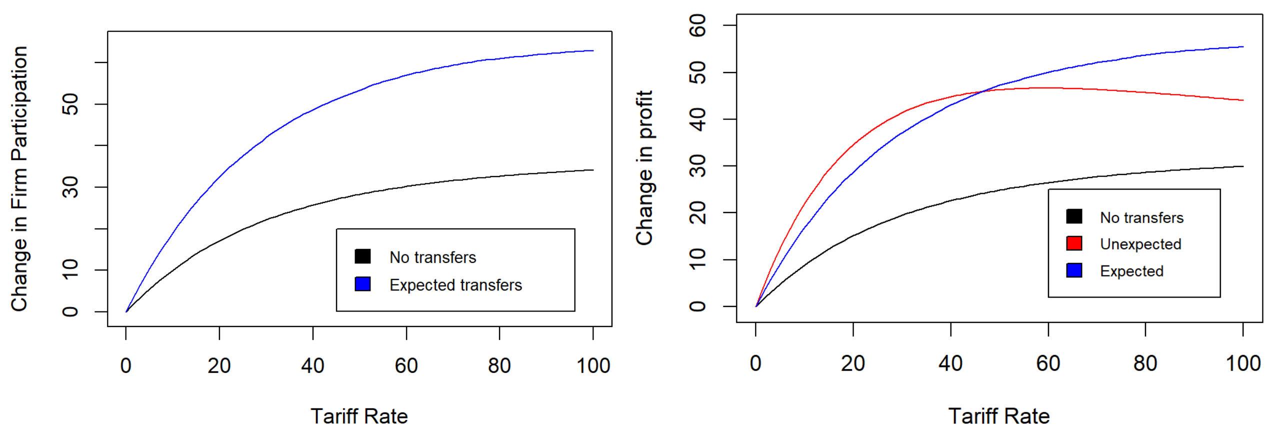


(a) Percent change in firm participation

(b) Percent change in total profit

Figure 2: Counterfactual outcomes as a function of intermediate tariff increases in both countries

falling.[7] Since input costs in both countries are now rising in tandem, the competitive disadvantage of paying higher input costs is less great at high values of intermediate goods tariff increase $\tilde{\tau}_u$, while Country 1 firms continue to receive the benefits of high transfer levels.



(a) Percent change in firm participation

(b) Percent change in total profit

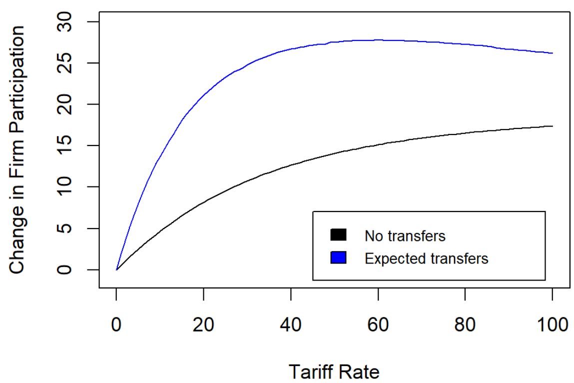
Figure 3: Counterfactual outcomes as a function of final goods tariff increases in Country 1 only

I next consider a scenario where Country 1 increases its final goods tariffs, while intermediate goods tariffs in Country 1 and all tariffs in Country 2 remain the same. As shown in Figure 3, without any simultaneously increasing tariffs in Country 2, Country 1 final goods producers benefit from decreased competition in their domestic market without any loss in exports. Counterfactual firm participation and profits are higher than their baseline counterparts even without transfer payments. In the presence of expected transfers, final goods tariffs have a larger effect on firm participation and profits than intermediate goods tariffs, with a 50% increase in final goods tariffs predicting a roughly 50% increase in firm participation and profits compared

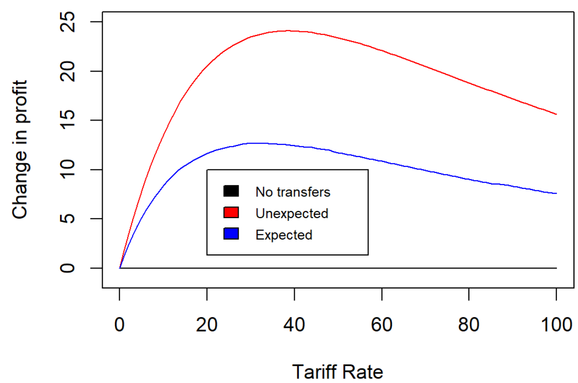
[7]If payments to final goods firms were symmetric across both countries, then X.

to a roughly 20% increase in Figure 1.

Interestingly, under a sufficiently high final goods tariff the increase in total firm profits under expected transfers begins to exceed the increase in profits under unexpected transfers. The influx of domestic firms induced to enter the market by expected transfers generally lowers total counterfactual profits compared to counterfactual profits with unexpected transfers, because these incoming firms are less productive than the firms already operating and their arrival undercuts market power. However, if tariffs and revenue are high enough, the influx of domestic firms is so large that it outweighs the loss of market power from each individual firm.



(a) Percent change in firm participation



(b) Percent change in total profit

Figure 4: Counterfactual outcomes as a function of final goods tariff increases in both countries

In Figure 4, I plot the results of increasing final goods tariffs by equal amounts in both countries. The model determines that reciprocal final goods tariffs, unlike reciprocal intermediate tariffs, do affect firm participation in a counterfactual without transfers. Because Country 2 initially imported 30% of Country 1 final goods ($\beta_{12} = 0.3$) but only 20% of Country 1 firms exported ($\phi_{12} = 0.2$), Country 2 is relatively reliant on Country 1 firms, and the exodus of exporting firms from Country 1 is outweighed by the influx of firms in Country 1 that produce for an exclusively domestic market. Profits, though, are still unchanged in the counterfactual without transfers because country symmetry ensures that the decrease in export profits exactly matches the increase in profits from the domestic market.

Comparing counterfactuals with transfer payments in Figure 3 and Figure 4, we see that final goods tariffs have a stronger positive effect on firm participation and profits if only one country raises tariffs. Unilateral final goods tariffs are purely beneficial in the model for final goods firms in the country initiating those tariffs, while under multilateral tariffs the increase in domestic sales and transfer payments accompanies a loss in exports. This relationship is different from the observed relationship in the intermediate tariff simulations, as counterfactual increases are higher in Figure 2 than in Figure 1.

Other curve behavior also differs. The firm participation and profit change curves are continually increasing in the tariff rate for Figure 3 but not Figure 4, while Figures 1 and 2 display the opposite pattern. Since unilateral final goods tariffs, unlike unilateral tariffs on intermediate goods, are purely beneficial for final goods firms in the country initiating those tariffs, the model does not predict any point at which the disadvantage of higher prices begins to exceed the advantage of transfers. If *both* countries impose tariffs, however, tariffs will reach a point where the disadvantage of lost exports outweighs the advantage of higher transfer payments. Nonetheless, given transfer payments counterfactual changes in firm participation m_1 and profits π_1 continue to be highly positive in both Figure 3 and Figure 4.

	Multi-stage model			Single-stage model		
	None	Unexpected	Expected	None	Unexpected	Expected
Firm participation	7.88%	7.88%	24.0%	7.87%	7.87%	37.1%
Domestic production	11.8%	11.1%	20.4%	17.7%	17.7%	20.2%
Domestic production (in levels)	12.4	12.4	14.3	12.4	12.4	14.1
Country 2 imports	-42.4%	-42.4%	-47.7%	-42.4%	-42.4%	-47.0%
Country 2 imports (in levels)	-12.4	-12.4	-14.3	-12.4	-12.4	-14.1
Total profits	0.0%	26.3%	14.7%	0.0%	26.4%	14.4%
Profits (in levels)	0.0	4.40	2.46	0.0	4.39	2.40

Table 1: Model simulation results with identical countries

Table 1 presents the counterfactual results of raising final goods tariffs, and only final goods tariffs, by 25% in a model with intermediate inputs and a model without intermediate inputs. Results show that the existence of multiple stages of production does not change how an increase in tariffs on final goods impacts equilibrium allocations. When comparing simulations with expected transfer payments, the increase in domestic production is *slightly* higher in the multi-stage model while the increase in profits is slightly lower.

Finally, I vary both tariffs simultaneously. Table 2 displays the results of a counterfactual where intermediate tariffs increase by 25% in both countries and final goods increase by 10% in both countries. All future simulations in this paper will feature these same counterfactual increases of 25% for intermediate tariffs and 10% for final goods tariffs.

Since bilateral intermediate goods tariffs without transfers do not affect firm participation, production, or profits, all effects on these allocations in column 6 come from the final goods tariffs. In the 'expected transfer' counterfactual, changes in firm participation, domestic production, imports and profits combine the effects of tariffs on intermediate goods with the effects of tariffs on final goods, so that when both tariffs increase the resulting effect on those allocations is greater than the effect of either tariff increase individually.

	Intermediate tariffs only		Final tariffs only		Both tariffs	
	None	Expected	None	Expected	None	Expected
Firm participation	0.00%	30.8%	4.67%	20.5%	4.67%	40.4%
Domestic production	0.00%	7.29%	10.5%	12.4%	10.5%	17.4%
Domestic production (in levels)	0.00	5.11	7.36	8.69	7.36	12.2
Country 2 imports	0.00%	-17.0%	-25.5%	-29.0%	-25.5%	-40.6%
Country 2 imports (in levels)	0.00	-5.11	-7.36	-8.69	-7.36	-12.2
Total profits	0.0%	29.0%	0.0%	8.45%	0.0%	33.0%
Profits (in levels)	0.0	9.66	0.0	2.82	0.0	11.0

	Unexpected transfer payments		
	Intermediate tariffs only	Final tariffs only	Both tariffs
Profits	43.2%	13.6%	56.7%
Profits (in levels)	14.4	4.53	18.9

Table 2: Model simulation results with identical countries and both tariffs applied

4.2 Input-Stage Parameters

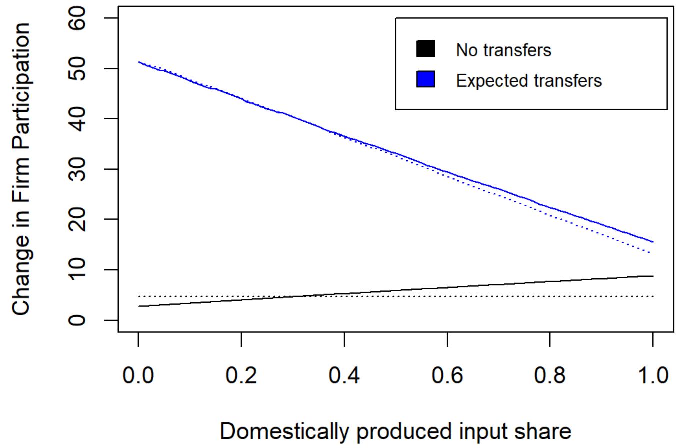
In this set of simulations, I analyze how counterfactual results vary depending on other parameters that appear in the input stage of production, namely input expenditure shares β_u and input elasticity of supply ε. While simulations in Section 4.1 looked at how model outcomes respond to different tariff counterfactuals, this section looks at how these counterfactual responses depend on underlying parameters of the model. All counterfactuals in this section feature reciprocal intermediate tariff increases of 25% and final goods tariff increases of 10%.

In Figure 5, I vary intermediate input shares within the two countries. The percentage of final good input cost spent on domestically produced inputs varies from 0 to 100% in Country 1 only (solid lines) and in both countries at once (dotted lines). The choice of β_u makes a large difference to counterfactual results, with increases in firm participation under expected transfers varying from 20% to 50% (about thirty percentage points) across the range of possible values. Unlike the tariff simulations in Section 4.1, results do not change much depending on whether expenditure shares change for both countries or only one.

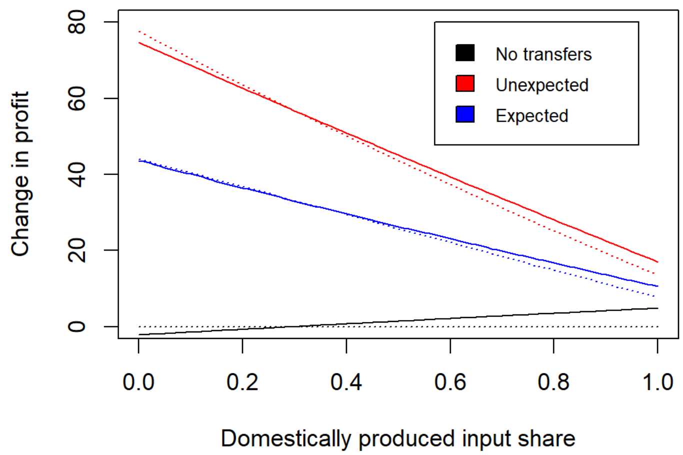
If firms do not receive any transfer payments, a higher domestic input expenditure share β_{11u} predicts a higher percentage increase in both firm participation and profits. A lower initial exposure to inputs means that input costs in Country 1 do not go up as much when it begins to put tariffs on its inputs. If firms do receive transfer payments, then change in firm participation $\tilde{m}$ and change in profits $\tilde{\pi}$ are decreasing in β_{11u} because Country 1 cannot collect as much tariff revenue if it is importing less; if $\beta_{11u} = 1$, for example, then Country 1 would not collect any tariff revenue from imports of intermediate goods.

In Figure 6, I perform a similar exercise with elasticities of input supply ε, alternatively varying ε from 0 to 7 for Country 1 only (solid lines) and for both countries at once (dotted lines).

If only Country 1's elasticity changes, then an increase in input supply elasticity ε predicts an increase



(a) Percent change in firm participation

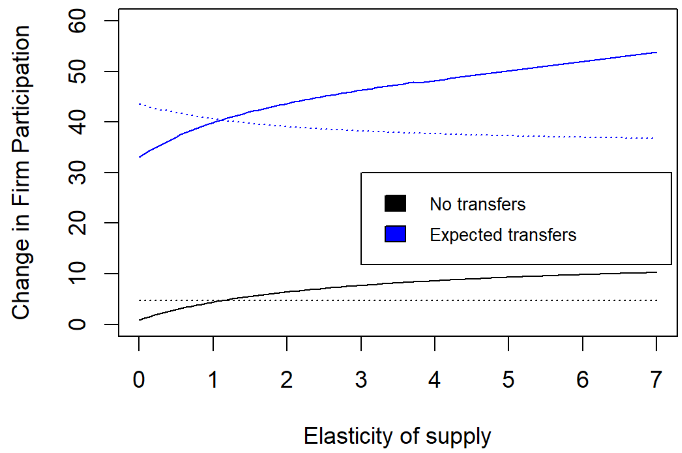


(b) Percent change in total profit

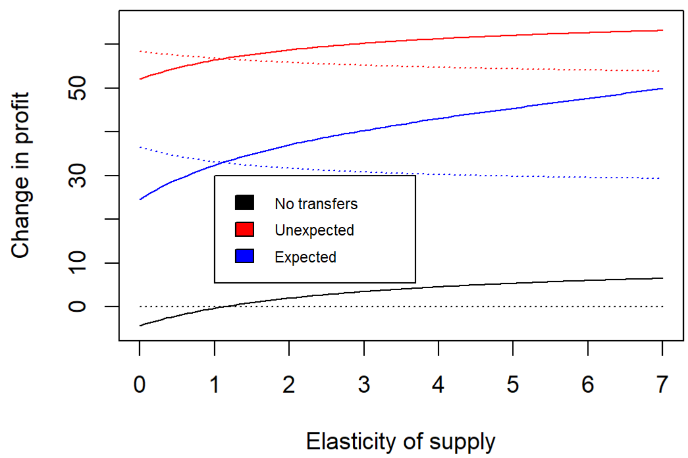
Figure 5: Counterfactual outcomes as a function of intermediate expenditure shares. Dotted lines represent counterfactuals where both countries are identical.

in counterfactual firm participation and profits. When prices rise in the counterfactual, Country 1 input producers ramp up production at a greater rate than Country 2 input producers because Country 1 input producers are more responsive to price increases. Because tariff increases are inducing each country's final goods producers to use more domestic inputs, Country 1 firms are more poised to benefit the higher the input supply elasticity is.

The range of possible counterfactual changes in Figure 6 is not as extensive as the range in Figure 5 or in Section 4.1, with the smaller range especially notable in the simulations where firms are identical. This result suggests that the choice of input supply elasticities does not affect modeling results as much as the choice of tariff counterfactual or the choice of input expenditure shares. Elasticity results also display diminishing marginal returns, so that, for example, an increase in the price elasticity of input supply from 1 to 2 would make a larger difference to counterfactual results than an increase from 5 to 6.



(a) Percent change in firm participation



(b) Percent change in total profit

Figure 6: Counterfactual outcomes as a function of input supply elasticities. Dotted lines represent counterfactuals where countries are identical.

5 Alternative Modeling Specifications

Users can adapt the model presented in Section 2 to reflect the economic environment for different countries, products, or industries. This section discusses a few of the possible model customizations that users may choose to implement in order to reflect specific market conditions.

5.1 Monopolistically Competitive Input Producers

Users may wish to model an industry environment where input producers are monopolistically competitive rather than perfectly competitive. Furthermore, the modeling of input producers as monopolistically competitive rather than perfectly competitive allows users to compute operating profits for input producers.

If input producers are monopolistically competitive, then they choose prices in a manner similar to that of final goods producers. Remembering that λ is the elasticity of substitution between final goods inputs, the price for an input exported from i to j is

$$P_{iju} = \frac{\lambda}{\lambda - 1}$$

where the cost of producing intermediate inputs is normalized to 1 and therefore does not appear in the price equation.[8] Because the right-hand side of this equation does not vary with time, $\tilde{P}_{iju} = 1 \ \forall i$. The computation of $\tilde{P}_{ju}$ becomes substantially simpler, with

$$\tilde{P}_{ju}^{1-\lambda_j} = \sum_{i=1}^{N} \beta_{iju} \tilde{\tau}_{iju}^{1-\lambda_j}$$

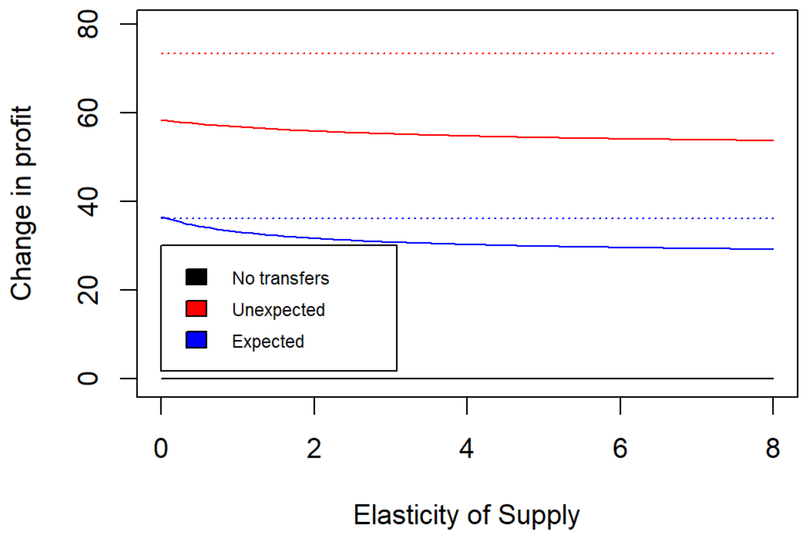
replacing (13). Counterfactual price increases are higher under monopolistic competition because the input firms' greater market power allows them to charge higher prices.

Figure 7 illustrates the dynamics of this alternative modeling specification, comparing counterfactual profits with monopolistically competitive input producers (dotted line) to counterfactual profits with perfectly competitive input producers as a function of the input supply elasticity ε (solid line). Although ε varies from 0 to 8, I set it to be the same across all origins and all destinations. Parameters are otherwise exactly as described in Section 4.2, with the exception of Figure 7b) where Country 1 imports 95% of its inputs by value.

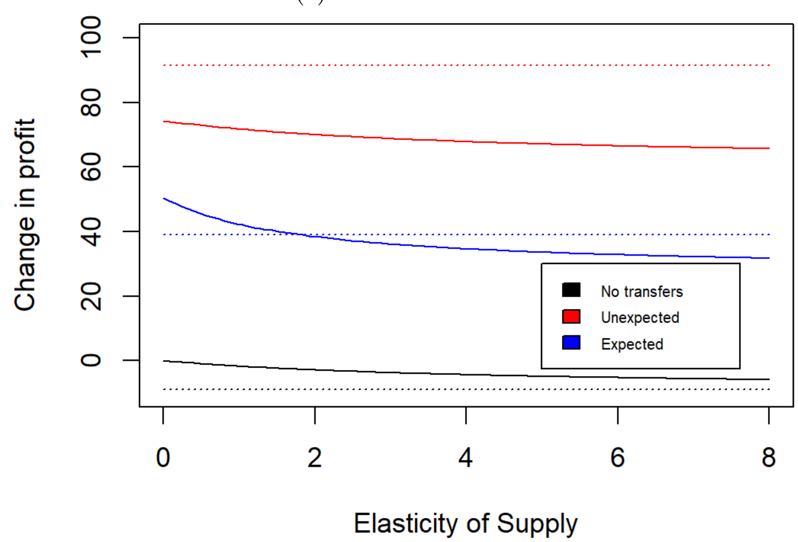
If firms do not receive transfer payments and the countries are identical, then the counterfactual change in profits is zero regardless of the structure for input firms. In Figure 7b), however, the higher prices final goods firms pay for their inputs causes the counterfactual decrease in profits to be higher when those firms source their inputs from monopolistically competitive firms. Higher input prices mean fewer inputs and less production, undercutting profits.

If firms receive transfer payments unexpectedly, final goods firms see higher profits with monopolistically competitive inputs. The higher input prices charged by monopolistically competitive firms depress final goods production but also result in higher tariff revenue, and hence higher transfers, because the tariffs are ad valorem. The relationship between input market structure and profit under expected transfers differs depending on input expenditure shares; if country 1 is more exposed to imported inputs, then higher imported input costs under monopolistic competition outweigh the benefits of receiving higher transfers, and perfect competition is preferable to monopolistic competition.

[8] Tariffs do not appear either because they already appear in the equations governing final good firms' demand for intermediate inputs.



(a) Identical countries



(b) $\beta_{1u} = (0.05, 0.95)$

Figure 7: Counterfactual outcomes with varying market structures for input producers. Dotted lines represent outcomes under monopolistic competition, while the solid lines represent outcomes under perfect competition.

5.2 Complementary Inputs

The model outlined in Section 2 features an elasticity of substitution of λ between inputs. Model users, however, may want to represent a scenario where both foreign and domestic inputs are necessary for production, and cannot be substituted for one another.[9] In such a scenario, the production function for final

[9] This situation is equivalent in mathematical terms to $\lambda \to 0$ in the limit.

goods would be

$$y_{ju} = \min\left\{y_{1ju}, \ldots, y_{Nju}\right\}$$

with baseline prices adjusting so that $\beta_{iju} = \frac{E_{iju}}{E_{ju}}\ \forall i$. The system of counterfactual equations described in Section 3.2 would change so that prices are given by

$$\tilde{P}_{ju} = \sum_{i=1}^{N} \beta_{iju} \tilde{P}_{iju} \tilde{\tau}_{iju}$$

and (16) simply becomes

$$\tilde{P}_{iju}^{\varepsilon_{ij}} = \frac{1}{\tilde{P}_{ju}} \frac{\sum_{i=1}^{N} \frac{\sigma_i - 1}{\sigma_i} E_i \beta_{ji} \tilde{P}_i^{\gamma_j} \tilde{P}_{ju}^{-\gamma_j}}{\sum_{j=1}^{N} \frac{\sigma_i - 1}{\sigma_i} E_i}. \tag{17}$$

Table 3 shows the results of running modeling simulations with $\lambda_1 = \lambda_2 = 0$ when other parameters across both countries are identical.

Allocation	Identical Firms		
	None	Unexpected	Expected
Firm participation	4.67%	4.67%	65.8%
Domestic production	10.5%	10.5%	21.9%
Domestic production (in levels)	7.36	7.36	15.3
Country 2 imports	-25.5%	-25.5%	-51.1%
Country 2 imports (in levels)	-7.36	-7.36	-15.3
Total profits	0.0%	107%	57.3%
Profits (in levels)	0.0	35.7	19.1

Table 3: Model simulation results with complementary inputs

The presence of complementary inputs raises input prices, because final good producers are unable to substitute between domestic and foreign inputs. When countries are symmetric (column 2), final good prices rise by the same percentage as input prices, so that production decisions are unchanged and column 2's results are the same as those of column 2 in Table 1. When the firms receive transfers, higher input prices mean that the transfer amounts increase relative to a scenario where $\lambda = 3$, resulting in high profit increases. The benefit of higher transfers outweighs the greater input costs that the firms must pay.

5.3 Additional Non-Manufactured Inputs

Model users may wish to model additional inputs that are not imported or manufactured. Labor is an example of one such input, and models representing agricultural industries frequently have other inputs such as land or water. The addition of additional inputs to the model is straightforward, as model users may simply incorporate them into the production equation for final goods. If we let $q_{1,\ldots,K}$ represent the K additional inputs that go into final production, then assuming the production of such inputs is perfectly competitive the supply equation for input k in country j would be

$$q_{jk} = P_{jk}^{\varepsilon_{jk}}$$

If we continue to assume CES aggregation then the production equation would be

$$y_{ju}^{\frac{\lambda_j - 1}{\lambda_j}} = \sum_{k=1}^{K} q_{jk}^{\frac{\lambda_j - 1}{\lambda_j}} + \sum_{i=1}^{N} \left(\frac{y_{iju}}{\tau_{iju}}\right)^{\frac{\lambda_j - 1}{\lambda_j}}$$

The q terms do not have tariff denominators because they are not imported.

Researchers may also vary the functional forms through which inputs appear in the production equation. For example, land, labor and water are probably not substitutes for one another, and for that reason a CES production function would not accurately represent them. A user may model CES substitution for manufactured inputs and complementarity for non-manufactured inputs through the following production equation:

$$y_{ju} = \min\{q_{j1}, \ldots, q_{jK}, h_j\}$$
$$h_j^{\frac{\lambda_j - 1}{\lambda_j}} = \sum_{i=1}^{N} \left(\frac{y_{iju}}{\tau_{iju}}\right)^{\frac{\lambda_j - 1}{\lambda_j}}$$

This production format allows final goods firms to substitute between manufactured inputs of various origins, but assumes that the aggregation of those inputs is complementary with other inputs such as land, water, etc. and cannot be substituted for one another. The following equations would relate input allocations:

$$
\begin{aligned}
y_{ju} &= h_j = q_{jk} \ \forall k \in \{1, K\} \\
y_{iju} &= h_j P_{jh}^{\lambda_j} P_{iju}^{-\lambda_j} \tau_{iju}^{1-\lambda_j} \\
P_{ju} &= \sum_{k=1}^{K} P_{jk} + P_{jh} \\
P_{jh}^{1-\lambda_j} &= \sum_{i=1}^{N} \left(P_{iju}\tau_{iju}\right)^{1-\lambda_j}
\end{aligned}
$$

6 Conclusion

In this paper, I build upon previous research by modeling lump-sum transfers of tariff revenue to firms in a model featuring two stages of production. Firms in the model that produce final goods use both domestically produced inputs and imported inputs, and tariff revenue includes revenue collected from tariffs on both imported inputs and imported final goods. The methodology can be applied to a wide variety of countries or industries, and allows for some versatility in how its users can model the production and use of intermediate inputs.

As part of the modeling, I perform a series of counterfactual simulations in which I estimate the effect of a change in tariffs given hypothetical parameter choices. While the paper does not apply observed data or model any real-life countries, it describes what types of data would generally be available and how a researcher might use this data to compute equilibrium solutions to the model. While the addition of multi-stage production does not measurably affect counterfactuals where only final goods tariffs increase, intermediate goods tariffs and final goods tariffs have large, and contrasting, effects on modeling outcomes. Moreover, input expenditure shares and, to a lesser extent, input supply elasticities also play an important role in determining model output.

Future modeling in this area could explore different ways to represent multi-stage production. For example, researchers could develop models where final production is in turn used as inputs when producing intermediate goods; these type of models are known as 'roundabout' production models. Alternatively, researchers interested in global value chains could embellish the model by adding additional stages of production, each with its own individualized tariff rate.

As with single-stage tariff transfer models, dynamics are an important factor that this paper does not consider. Dynamic partial equilibrium models could represent situations in which the effect of transfer payments is not instantaneous, or alternatively situations where firms receive the transfer payments at different points in time. However, any modeling that features both dynamic time horizons and multiple stages of production would quickly become unwieldly and difficult to execute. In the end, a researcher's biggest contribution to this class of models may be the pioneering of a modeling technique that allows users to address relevant areas of inquiry while keeping things *simple*.

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7 Appendix

This appendix discusses how to solve the model in more detail than the detail given in Section 3.2. Since Phillips (2025) derived equations for a similar model, discussions focus on the equations that were not derived in that earlier paper, and any such equations appear here without further elucidatory comment.

I begin with the derivation of (13). From (2) we know that

$$\beta_{iju} = \frac{E_{iju}}{E_{ju}} = (\frac{P_{ju}}{P_{iju}\tau_{iju}})^{\lambda_j - 1}$$

and hence

$$(\frac{\hat{P}_{ju}}{\hat{P}_{iju}\hat{\tau}_{iju}})^{\lambda_j - 1} = (\frac{\tilde{P}_{ju}P_{ju}}{\tilde{P}_{iju}P_{iju}\tilde{\tau}_{iju}\tau_{iju}})^{\lambda_j - 1} = \beta_{iju}(\frac{\tilde{P}_{ju}}{\tilde{P}_{iju}\tilde{\tau}_{iju}})^{\lambda_j - 1}$$

If I write out (3) in the counterfactual and divide both sides by $\hat{P}_{ju}^{1-\lambda_i}$, then I get

$$1 = \sum_{i=1}^{N}(\frac{\hat{P}_{iju}\hat{\tau}_{iju}}{\hat{P}_{ju}})^{1-\lambda_j}$$
$$\Rightarrow 1 = \sum_{i=1}^{N}\beta_{iju}(\frac{\tilde{P}_{ju}}{\tilde{P}_{iju}\tilde{\tau}_{iju}})^{\lambda_j - 1}$$

Moving $\tilde{P}_{ju}$ through to the left-hand side leads me to (13).

I derive (15) by incorporating (14) into (13). Rearranging (14) leads to an equation for $\tilde{P}_{jiu}$:

$$\tilde{P}_{iju} = \left(\tilde{\tau}_{iju}^{-\lambda_j}\tilde{P}_{jju}^{\lambda_j + \varepsilon_{ij}}\right)^{\frac{1}{\lambda_j + \varepsilon_{ij}}} \tag{18}$$

and then I can simply replace $\tilde{P}_{jiu}$ in (13) with (17). The resulting equation is messy but the algebraic process is simple, and the equation simplifies if users make the assumption $\varepsilon_{ii} = \varepsilon_{ji} = \varepsilon\ \forall i \forall j$.

(16) derives from market clearing conditions (8) and (9), first-order condition (2), and (5), which I rearrange by dividing both sides by E_i and replacing the fraction $\frac{E_{ji}}{E_i}$ with β_{ji}. Combining (8) and (9) and expressing variables as fractional changes, I know that

$$\tilde{P}_{iju}^{\varepsilon_{ij}} = \frac{\hat{y}_{ji} + \hat{y}_{jj}}{y_{ji} + y_{jj}}\tilde{P}_{ju}^{\lambda_j}(\tilde{\tau}_{iju}\tilde{P}_{iju})^{-\lambda_j}$$

and then I can use consumers' first-order conditions to replace equilibrium quantities denoted by y. However, because (5) and parameters β are denoted in values, I ultimately want to express equations in values rather than quantities. While the *quantity* of aggregated inputs must equal the quantity of production, the final good firms' markups differentiate the *value* of aggregated inputs from the value of production. I therefore know that

$$\begin{aligned}
y_{ju}P_{ju} &= \sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_{ji} \\
&= \sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}\mu_j\int_{\overline{x}_{ji}}^{\infty}P_{ji}(x)^{1-\sigma_i}E_iP_i^{\sigma_i-1}dF_j(x) \\
&= \mu_j\sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_iP_i^{\sigma_i-1}\int_{\overline{x}_{ji}}^{\infty}\Big(\frac{\tau_{ji}\sigma_iP_{ju}}{(\sigma_i-1)x}\Big)^{1-\sigma_i}\gamma_jx^{-\gamma_j-1}dx \\
&= \mu_j\sum_{i=1}^{N}\Big(\frac{\sigma_i}{\sigma_i-1}\Big)^{-\sigma_i}E_iP_i^{\sigma_i-1}P_{ju}^{1-\sigma_i}\tau_{ji}^{1-\sigma_i}\gamma_j\frac{\overline{x}_{ji}^{\sigma_i-1-\gamma_j}}{\gamma_j-(\sigma_i-1)}
\end{aligned}$$

I can use (5) to replace most of the items on the right-hand side of this equation, giving me

$$y_{ju}P_{ju} = \sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_i\beta_{ji}$$

The algebraic process for counterfactual allocation $\hat{y}_{ju}\hat{P}_{ju}$ is similar:

$$\begin{aligned}
\hat{y}_{ju}\hat{P}_{ju} &= \sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}\hat{E}_{ji} \\
&= \mu_j\sum_{i=1}^{N}\Big(\frac{\sigma_i}{\sigma_i-1}\Big)^{-\sigma_i}E_i\hat{P}_i^{\sigma_i-1}\hat{P}_{ju}^{1-\sigma_i}\hat{\tau}_{ji}^{1-\sigma_i}\gamma_j\frac{\hat{\overline{x}}_{ji}^{\sigma_i-1-\gamma_j}}{\gamma_j-(\sigma_i-1)}
\end{aligned}$$

Using a process described in Phillips (2025), I can replace $\hat{\overline{x}}_{ij}$ with β_{ij} and prices, which leads to the equation

$$\hat{y}_{ju}\hat{P}_{ju} = \sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_i\beta_{ji}\tilde{P}_i^{\gamma_j}(\tilde{\tau}_{ji}\tilde{P}_{ju})^{-\gamma_j}$$

So then

$$\tilde{p}_{iju}^{\varepsilon_{ij}} = \tilde{P}_{ju}^{\lambda_j}(\tilde{\tau}_{iju}\tilde{P}_{iju})^{-\lambda_j}\frac{1}{\tilde{P}_{ju}}\frac{\sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_i\beta_{ji}\tilde{P}_i^{\gamma_j}\tilde{P}_{ju}^{-\gamma_j}\tilde{\tau}_{ji}^{-\gamma_j}}{\sum_{i=1}^{N}\frac{\sigma_i-1}{\sigma_i}E_i}$$