STRUCTURAL EQUATIONS FOR PE MODELS IN GROUP 3 (FIRM HETEROGENEITY, NEW ENTRY, AND INTELLECTUAL PROPERTY)

David Riker and Samantha Schreiber

U.S. International Trade Commission, Office of Economics

May 2019

Abstract

This paper presents the structural equations for the third group of industry-specific simulation models of changes in trade policy that are available for download on the USITC's PE Modeling Portal at `https://www.usitc.gov/data/pe_modeling/index.htm`.

 Please address correspondence to david.riker@usitc.gov.

1 Introduction

One of the spreadsheet models incorporates fixed costs and firm heterogeneity. The second model allows for the entry of imports from a new source. The third values the monopoly profits created by the protection of intellectual property rights. The fourth examines global incentives to innovate.

2 Model with Fixed Costs and Firm Heterogeneity

The first model is a two-country Melitz (2003) model of trade with firm heterogeneity. The model adopts useful simplifications and distributional assumptions in Helpman, Melitz and Yeaple (2004) and Chaney (2008).[1] Within the industry, there is a continuum of firms supplying differentiated products, with constant elasticity of substitution σ. The firms vary in their unit labor requirements. Firm-specific productivity has a Pareto distribution with shape parameter γ.[2] There is a fixed cost of production f_D and an incremental fixed cost of exporting f_X.

D is the aggregate value of domestic shipments in the market, integrated over the mass n_D of domestic suppliers.[3]

$$D = k \left(\frac{n_D}{n_D + n_F \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}} \right) \tag{1}$$

M is the aggregate value of imports into the market, integrated over the mass n_F of foreign suppliers. k is total industry expenditures in the market, and τ is a variable trade cost on

[1] Khachaturian and Riker (2016) provides a step-by-step derivation of the model, for an extended version that includes FDI.

[2] The model adopts the standard assumption in the literature that $\gamma > \sigma - 1$. DiGiovanni, Levchenko and Rancière (2011) is a source for industry-specific econometric estimates of these parameter values.

[3] Following Chaney (2008), the model assumes that the number of firms that participate in the market is endogenously determined but the numbers of potential market participants, n_D and n_F, are exogenous.

imports.

$$M = k \left(\frac{n_F \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}}{n_D + n_F \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}} \right) \tag{2}$$

Equation (3) is the ratio of the value of imports to the value of domestic shipments.

$$\frac{M}{D} = (\tau)^{-\gamma} \left(\frac{n_F}{n_D} \right) \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1} \tag{3}$$

Next, we define $Z_0 = \left(\frac{n_F}{n_D} \right) \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}$. The model calibrates Z_0 based on the ratio of the value of imports to the value of domestic shipments in the initial equilibrium and initial trade costs.

$$Z_0 = \left(\frac{M_0}{D_0} \right) \ (\tau_0)^{\gamma} \tag{4}$$

τ_0 is the initial trade cost, M_0 is the initial value of imports, and D_0 is the initial value of domestic shipments. An increase in the fixed cost of exporting f_X decreases Z and relative expenditure on imports, and an increase in the fixed cost of domestic production f_D increases Z and relative expenditure on imports.

$$Z = Z_0 \ \left(1 + \left(\frac{-\gamma}{\sigma - 1} + 1 \right) \left(\left(\frac{f_X - f_{X0}}{f_{X0}} \right) - \left(\frac{f_D - f_{D0}}{f_{D0}} \right) \right) \right) \tag{5}$$

The model simulates the effects of changes in the fixed costs (f_X and f_D) and the variable trade cost (τ) on the value of imports (M) and the value of domestic shipments (D), based on (6) and (7).

$$D = D_0 \ \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}} \right) \tag{6}$$

$$M = M_0 \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}}\right) \left(\frac{Z(\tau)^{-\gamma}}{Z_0(\tau_0)^{-\gamma}}\right) \tag{7}$$

3 Model with Entry of a New Source of Imports

The second model addresses the entry of new sources of imports.[4] There are initially three sources of supply to the market, one domestic source (x) and two foreign sources (y and z). Consumers have CES preferences, with elasticity of substitution σ and a price elasticity of total industry demand equal to η. There is perfect competition in the market, and the supply from all three sources is perfectly elastic (i.e., there are no capacity constraints on production), so $p_j = c_j$ for $j \in \{x, y, z\}$.

Equation (8) is the original CES price index for the market, and (9) is the initial equilibrium demand for the product from source $j \in \{x, y, z\}$.

$$P_0 = \left((p_{x0})^{1-\sigma} + b_y \ (p_{y0} \ \tau_{y0})^{1-\sigma} + b_z \ (p_{z0} \ \tau_{z0})^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{8}$$

$$q_{j0} = k \ (P_0)^{\sigma+\eta} \ (p_{j0} \ \tau_{j0})^{-\sigma} \ b_j \tag{9}$$

Equations (10) through (12) calibrate the three demand parameters to initial expenditures and tariff rates, normalizing initial prices to one without loss of generality.

$$b_y = \left(\frac{v_{y0}}{v_{x0}}\right) (\tau_{y0})^{\sigma-1} \tag{10}$$

$$b_z = \left(\frac{v_{z0}}{v_{x0}}\right) (\tau_{z0})^{\sigma-1} \tag{11}$$

[4] Riker (2019) presents an extended version of this model.

$$k = v_{x0} \ \left(1 + b_y \ (\tau_{y0})^{1-\sigma} + b_z \ (\tau_{z0})^{1-\sigma}\right)^{\frac{-\sigma-\eta}{1-\sigma}} \tag{12}$$

Entry leads to a fourth source of supply, entrant e. The model assumes that source z is an appropriate reference group, meaning that the production costs and perceived quality of the products of the new entrant (c_e and b_e) are the same as those of the reference group (c_z and b_z). This is not a model of endogenous entry; it is modeling the economic impact conditional on entry.[5]

Equation (13) is the new equilibrium CES price index that includes source e.

$$P = \left((p_x)^{1-\sigma} + b_y \ (p_y \ \tau_y)^{1-\sigma} + b_z \ (p_z \ \tau_z)^{1-\sigma} + b_e \ (p_e \ \tau_e)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{13}$$

Since supply from each source is perfectly elastic, $p_j = p_{j0} = c_j$ for $j \in \{x, y, z, e\}$. Equation (14) is the new equilibrium quantity from source j.

$$q_j = k \ (P)^{\sigma+\eta} \ (p_j \ \tau_j)^{-\sigma} \ b_j \tag{14}$$

Finally, (15) is the new equilibrium market share of source j.

$$m_j = \frac{b_j \ (p_j \ \tau_j)^{1-\sigma}}{(p_x)^{1-\sigma} + b_y \ (p_y \ \tau_y)^{1-\sigma} + b_z \ (p_z \ \tau_z)^{1-\sigma} + b_e \ (p_e \ \tau_e)^{1-\sigma}} \tag{15}$$

The model can be used to simulate the entry of a new source of imports, for example due to the reduction or removal of a prohibitive tariff on imports from the new source.

[5]The issue of conditional entry is discussed in detail in Riker (2019).

4 Model of the Value of a Monopoly Created by Protecting Intellectual Property Rights

In the third model, there is a linear demand curve for the products of the market.[6]

$$Q = a - b\ P \tag{16}$$

Equation (17) is the initial price elasticity of total industry demand.

$$\eta_0 = \frac{\partial Q_0}{\partial P0}\ \frac{P_0}{Q_0}\ = -b\ \left(\frac{P_0}{Q_0}\right) \tag{17}$$

There is initially perfect competition, because intellectual property rights (IPRs) are not protected, so price is equal to marginal cost. Profits are competed to zero by infringing or imitating firms. Equations (18) through (20) calibrate marginal costs and the two parameters of the demand curve based on the initial equilibrium price and quantity, P_0 and Q_0.

$$c = P_0 \tag{18}$$

$$b = \eta_0 \left(\frac{Q_0}{P_0}\right) \tag{19}$$

$$a = Q_0\ \ (1 + \eta_0) \tag{20}$$

The protection of IPRs creates a monopoly in the market, and there is a new market equilibrium. Equation (21) is monopoly profits, in terms of the new monopoly price P_m and

[6]It can be problematic to assume a constant elasticity demand curve in a monopoly model, since the profit-maximizing price will be infinite for an elasticity of one or below in absolute value. For this reason, monopoly models often assume a linear demand curve.

quantity Q_m.

$$\pi_m = (P_m - c)\ Q_m \tag{21}$$

Equation (22) is the first order condition for monopoly pricing.

$$\frac{\partial \pi_m}{\partial P_m} = a - 2\ b\ P_m + b\ c = 0 \tag{22}$$

This first order condition implies the monopoly price in (23), the percent change in price in (24), and the percentage change in quantity in (25).

$$P_m = P_0\ \left(\frac{1 + 2\ \eta_0}{2\ \eta_0}\right) \tag{23}$$

$$\frac{P_m - P_0}{P_0} = \frac{1}{2}\left(\frac{1}{\eta_0}\right) \tag{24}$$

$$\frac{Q_m - Q_0}{Q_0} = -\left(\frac{1}{2}\right) \tag{25}$$

Finally, (26) is the value of monopoly profits at the new equilibrium, as a function of total industry revenues in the initial equilibrium, $R_0 = P_0\ Q_0$.

$$\pi_m = \left(\frac{1}{4\ \eta_0}\right)\ R_0 \tag{26}$$

5 Model of Trade and Innovation

The fourth model addresses how the protection of IPRs affects incentives to innovate. It is based on models of trade, product diversity, and monopolistic competition in Krugman (1980). It is a simpler, static, partial equilibrium version of the model with innovation

and horizontal differentiation in Grossman and Helpman (1989). As we note below, the same model can be applied – with specific modifications to the model inputs – to address innovation that creates vertical differentiation, by reducing production costs or increasing product quality.

Within each industry, consumers have symmetric CES preferences with elasticity of substitution σ. There are Cobb-Douglas preferences between industries, which implies that the price elasticity of total industry demand is equal to -1.

There is a fixed cost to invent a new variety, f, and constant marginal costs of production c. The "blueprint" for each variety is non-rival in its use in different countries, so there are global scale economies to innovation, as long as the returns to innovation are ensured by the protection of IPRs.

There are a number of national markets, indexed by j, in which IPRs are protected in the initial equilibrium. In each market, there is a continuum of varieties. Each firm prices at a constant mark-up over marginal cost. Equation (27) are initial profits in market j.

$$\pi_j = \frac{1}{\sigma} R_j \tag{27}$$

R_j are initial revenues in country j. The model assumes that laws that protect IPRs create a monopoly in the variety that would otherwise not exist. Unrestricted imitation and infringement would drive the mark-up to zero and eliminate the incentive to develop the additional variety. Equation (28) is the initial number of varieties, N_0.

$$N_0 = \frac{1}{\sigma f} \sum_j R_j \tag{28}$$

With the additional protection of IPRs in country k, the equilibrium number of varieties will increase to N.

$$N = N_0 + \frac{R_k}{\sigma f} = \frac{1}{\sigma f}\left(\sum_j R_j + R_k\right) \tag{29}$$

Equation (30) is the percent change in the total number of product varieties developed. This measure of innovation increases in proportion to the size of the global sum of the IPR-protected national markets.

$$\frac{N - N_0}{N_0} = \frac{R_k}{\sum_j R_j} \tag{30}$$

Equation (31) is the simulated change in the value of innovations from protecting IPRs in the additional markets.

$$f\ \Delta N = \frac{1}{\sigma} R_k \tag{31}$$

If innovation leads to cost reductions then the IPR-protected mark-up is determined by the cost advantage of the technology leader over non-infringing imitators, rather than the reciprocal of the elasticity of substitution. The model can be applied to this alternative scenario by changing the model inputs. If innovation leads to quality reductions, then the mark-up would be based on the quality step.[7]

References

Chaney, T. (2008). Distorted Gravity: The Intensive and Extensive Margins of International Trade, *American Economic Review* **98**(4): 1707–1721.

DiGiovanni, J., Levchenko, A. A. and Rancière, R. (2011). Power Laws in Firm Size and Openness to Trade: Measurement and Implications, *Journal of International Economics*

[7]Examples of models with vertical differentiation include Grossman and Helpman (1990), Grossman and Helpman (1991a), and Grossman and Helpman (1991b).

85: 45–52.

Grossman, G. and Helpman, E. (1989). Product Development and International Trade, *Journal of Political Economy* **97**(6): 1261–1283.

Grossman, G. and Helpman, E. (1990). Trade, Innovation, and Growth, *American Economic Review Papers and Proceedings* **80**(2): 86–91.

Grossman, G. and Helpman, E. (1991a). Quality Ladders and Product Cycles, *Quarterly Journal of Economics* **106**(2): 557–586.

Grossman, G. and Helpman, E. (1991b). Quality Ladders in the Theory of Growth, *Review of Economic Studies* **58**(1): 43–61.

Helpman, E., Melitz, M. J. and Yeaple, S. R. (2004). Exports versus FDI with Heterogenous Firms, *American Economic Review* **94**(1): 300–316.

Khachaturian, T. and Riker, D. (2016). A Multi-Mode Partial Equilibrium Model of Trade in Professional Services, *U.S. International Trade Commission Economics Working Paper* **No. 2016-11-A**.

Krugman, P. (1980). Scale Economies, Product Differentiation, and the Pattern of Trade, *American Economic Review* **70**(5): 950–959.

Melitz, M. J. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica* **71**(6): 1695–1725.

Riker, D. (2019). An Industry-Specific Model with New Entry of Subject Imports, *U.S. International Trade Commission Economics Working Paper* **No. 2019-02-A**.