Translog Partial Equilibrium Model of Trade Policy Changes

Samantha Schreiber

ECONOMICS WORKING PAPER SERIES
Working Paper 2019–07–A

U.S. INTERNATIONAL TRADE COMMISSION
500 E Street SW
Washington, DC 20436

July 2019

Office of Economics working papers are the result of ongoing professional research of USITC Staff and are solely meant to represent the opinions and professional research of individual authors. These papers are not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners. Working papers are circulated to promote the active exchange of ideas between USITC Staff and recognized experts outside the USITC and to promote professional development of Office Staff by encouraging outside professional critique of staff research. The author thanks David Riker for helpful comments and related research collaboration. Please address correspondence to samantha.schreiber@usitc.gov.

Translog Partial Equilibrium Model of Trade Policy Changes
Samantha Schreiber
Office of Economics Working Paper 2019–07–A
July 2019

Abstract

We present a sector-specific partial equilibrium model that quantifies the economic impacts of a tariff change on prices and quantities. The model uses transcendental logarithmic (translog) preferences, originally proposed by Christensen, Jorgenson and Lau (1975) and made popular in recent times by Feenstra, instead of the commonly-used constant elasticity of subtitution (CES) preferences.

Samantha Schreiber, Office of Economics
samantha.schreiber@usitc.gov

1 Introduction

Constant elasticity of substitution (CES) is a common demand functional form assumption used in trade policy models with one constant elasticity parameter that characterizes substitutability across all sources of supply[1]. This assumption has a number of attractive properties such as limited data requirements and algebraic ease. However, one substitution elasticity may be too restrictive if there are large substitutability differences across goods. In this paper we describe an alternative to CES, the "transcendental logarithmic" or translog model, that allows for greater variety of substitution patterns across pairs of goods.

The translog model was first proposed by Christensen, Jorgenson and Lau (Christensen, Jorgenson and Lau, 1975) but made popular in recent times by Feenstra (Bergin and Feenstra, 2000; Bergin and Feenstra, 2001; Feenstra and Weinstein, 2017). The model begins with a quadratic, logarithmic indirect utility function using expenditure-normalized prices[2]. By Roy's Identity, we derive a system of budget share equations that represent consumer demand for each good. There are own- and cross-price demand elasticities for each pair of goods, partially restricted in the model to align with theoretical underpinnings.

First we will present the theory behind the translog model in section 2. In section 3, we describe popular restrictions placed on the model to arrive at a solution. Then we show illustrative simulation results with different model inputs in section 4. Section 5 offers areas of further research and concludes.

2 Model

Denote the price of each good $i \in \{1, 2, ..n\}$ as p_i. There are n total sources of supply, including both domestic and imported varieties. Define M as total expenditures, α_i as a

[1] Or two elasticity parameters for a nested-CES specification, one for domestic goods and an import aggregate, and one for all imported goods.

[2] The use of expenditure-normalized prices (p_j/M) imposes homogeneity.

demand shift parameter, t_i as the tariff rate, and q_i as the quantity of each good i. Define γ_{ij} to be a utility parameter representing the coefficient on the interaction of log prices for $i, j \in \{1, 2, ...n\}$. The translog indirect utility function $log\ V$ takes the form:

$$logV = \alpha_0 + \sum_i \alpha_i\ log(\frac{p_i}{M}) + \frac{1}{2} \sum_i \sum_j \gamma_{ij}\ log(\frac{p_i}{M})\ log(\frac{p_j}{M}) \tag{1}$$

By Roy's identity, the budget share for the ith good is:

$$\frac{p_i\ q_i}{M} = -\ \frac{\partial logV}{\partial logp_i} \bigg/ \frac{\partial logV}{\partial logM} \tag{2}$$

Partially differentiating this form of the indirect utility function:

$$\frac{\partial logV}{\partial logp_i} = \alpha_i + \sum_j \gamma_{ij}\ log\frac{p_j}{M} \tag{3}$$

$$-\ \frac{\partial logV}{\partial logM} = \sum_k \left(\alpha_k + \sum_j \gamma_{kj}\ log\frac{p_j}{M}\right) \tag{4}$$

Define $\alpha_M = \sum_k \alpha_k$ and $\gamma_{Mj} = \sum_i \gamma_{ij}$ for ease of notation. This gives us budget share equations for each i:

$$\frac{p_i q_i}{M} = \frac{\alpha_i + \sum_j \gamma_{ij} log(\frac{p_j}{M})}{\alpha_M + \sum_j \gamma_{Mj} log(\frac{p_j}{M})} \tag{5}$$

We use the γ parameters to calculate the uncompensated price elasticities in this model as:

$$\eta_{ij} = -\delta_{ij} + \frac{\frac{\gamma_{ij}}{w_i} - \sum_j \gamma_{ij}}{-1 + \sum_k \gamma_{Mk} log(\frac{p_k}{M})} \tag{6}$$

where δ_{ij} is the kronecker delta, equal to one if $i = j$ and zero otherwise, and w_i is the budget share of good i. On the supply side, let ϵ_i be the constant price elasticity of supply, and suppose b_i is a supply parameter. Then the supply equations for each source i are:

$$q_i = b_i \; p_i^{\epsilon_i} \tag{7}$$

This gives us i budget share equations and i price unknowns. The user solves the model by supplying initial expenditure data for each good i, specifying demand parameters γ_{ij}, supply elasticity parameters ϵ_i and the current and new tariff rate policy shock. The rest of the parameters are calibrated to the initial data inputs.

3 Restrictions

The first restriction for the translog model, required by theory, is to impose symmetry in the γ demand parameters ($\gamma_{ij} = \gamma_{ji}$). For n goods in the model, this limits the number of γ inputs to $\frac{n(n+1)}{2}$. An additional restriction popular in the literature (Bergin and Feenstra, 2001) is to impose an additivity constraint so that $\sum_j \gamma_{ij} = 0$. This further limits the number of demand parameter inputs to $\frac{n(n-1)}{2}$. Third, also common in the literature (Christensen et al., 1975; Bergin and Feenstra, 2001; Holt and Goodwin, 2009) is to normalize the α_i demand parameters so that $\sum_i \alpha_i = -1$. With this restriction, you can use $(n-1)$ budget share equations to calibrate the α_i's to initial expenditure data inputs[3].

For three sources of supply (domestic production, subject imports, and non-subject imports), there are nine γ demand parameters and three α demand parameters. Imposing the first restriction, $\gamma_{ij} = \gamma_{ji}$ eliminates three of the nine parameters, and imposing the second restriction brings the number of free γ parameters to three. Imposing the third restriction, $\sum_i \alpha_i = -1$, and using $n-1$ initial budget share conditions, the modeler can calibrate all three α parameters with initial data inputs.

[3] As pointed out by Christensen, Jorgenson and Lau, there are more parameters than necessary to qualify as a second-order locally flexible functional form.

4 Illustrative Simulations with Three Goods

In this section, we assume there are three sources of supply: domestic production d, subject imports s, and non-subject imports n. Using the equations from section 2 and restrictions from section 3, the budget share equations and supply equations are:

$$\frac{p_d\ q_d}{M} = \frac{\alpha_d + \gamma_{ds}\ log\frac{p_s\ t_s}{M} + \gamma_{dn}\ log\frac{p_n}{M} + \gamma_{dd}\ log\frac{p_d}{M}}{(\gamma_{ds} + \gamma_{ns} + \gamma_{ss})\ log\frac{p_s\ t_s}{M} + (\gamma_{dn} + \gamma_{nn} + \gamma_{sn})\ log\frac{p_n}{M} + (\gamma_{dd} + \gamma_{nd} + \gamma_{sd})\ log\frac{p_d}{M} - 1} \tag{8}$$

$$\frac{p_s\ t_s\ q_s}{M} = \frac{\alpha_s + \gamma_{ss}\ log\frac{p_s\ t_s}{M} + \gamma_{sn}\ log\frac{p_n}{M} + \gamma_{sd}\ log\frac{p_d}{M}}{(\gamma_{ds} + \gamma_{ns} + \gamma_{ss})\ log\frac{p_s\ t_s}{M} + (\gamma_{dn} + \gamma_{nn} + \gamma_{sn})\ log\frac{p_n}{M} + (\gamma_{dd} + \gamma_{nd} + \gamma_{sd})\ log\frac{p_d}{M} - 1} \tag{9}$$

$$\frac{p_n\ q_n}{M} = \frac{\alpha_n + \gamma_{ns}\ log\frac{p_s\ t_s}{M} + \gamma_{nn}\ log\frac{p_n}{M} + \gamma_{nd}\ log\frac{p_d}{M}}{(\gamma_{ds} + \gamma_{ns} + \gamma_{ss})\ log\frac{p_s\ t_s}{M} + (\gamma_{dn} + \gamma_{nn} + \gamma_{sn})\ log\frac{p_n}{M} + (\gamma_{dd} + \gamma_{nd} + \gamma_{sd})\ log\frac{p_d}{M} - 1} \tag{10}$$

$$q_d = b_d p_d^{\epsilon_d} \tag{11}$$

$$q_s = b_s p_s^{\epsilon_s} \tag{12}$$

$$q_n = b_n p_n^{\epsilon_n} \tag{13}$$

We impose all three restrictions described in the third section. Table 1 reports model results under two different sets of elasticity values. The first section of the table shows model input

Table 1: Illustrative Simulations of Translog Model

	Simulation 1	Simulation 2
Initial Market Share of Domestic Goods	70%	70%
Initial Market Share of Subject Imports	20%	20%
Initial Market Share of Non-Subject Imports	10%	10%
New Tariff Rate	10%	10%
Price elasticity of supply, domestic good	2	2
Price elasticity of supply, imports	10	10
Gamma, domestic and subject import interaction coefficient	-0.25	-0.4
Gamma, domestic and non-subject import interaction coefficient	-0.1	-0.3
Gamma, subject and non-subject import interaction coefficient	-0.003	0
Uncompensated price elasticity, domestic good	-1.5	-2
Uncompensated price elasticity, subject import good	- 2.25	-3
Uncompensated price elasticity, non-subject import good	-2.05	-4
Cross-price elasticity, domestic response to Δ in p_s	0.35	0.6
Cross-price elasticity, domestic response to Δ in p_n	0.15	0.4
Cross-price elasticity, subject-import response to Δ in p_d	1.24	2
Cross-price elasticity, non-subject import response to Δ in p_d	1.03	3
% change in domestic producer price	0.79%	1.08%
% change in subject import producer price	-1.69%	-2.08%
% change in subject import consumer price	8.14%	7.7%
% change in non-subject import producer price	0.08%	0.23%
% change in quantity of domestic shipments	1.58%	2.17%
% change in quantity of subject imports	-15.67%	-18.96%
% change in quantity of non-subject imports	0.83%	2.3%

values. The second section reports the calculated uncompensated price elasticity values from the initial model inputs. The third section of the table presents the model results.

5 Conclusion

The translog model, made popular recently in the literature, is a departure from the traditional constant elasticity of substitution (CES) assumption typically used in partial equilibrium trade models. The translog model allows for greater variety in substitutability across sources of supply, but also requires that the modeler supply additional parameter values which may be difficult to obtain. This model should be combined with an econometric estimation of parameter inputs to generate theory-consistent, data-driven results.

References

Bergin, P. R. and Feenstra, R. C. (2000). Pricing-to-market, staggered contracts, and real exchange rate persistence, *Journal of International Economics* **54**: 333–359.

Bergin, P. R. and Feenstra, R. C. (2001). Staggered price setting, translog preferences, and endogenous persistence, *Journal of Monetary Economics* **45**.

Christensen, L. R., Jorgenson, D. W. and Lau, L. J. (1975). Transcendental logarithmic utility functions, *The American Economic Review* **65**.

Feenstra, R. C. (2003). A homothetic utility function for monopolistic competition models, without constant price elasticity, *Economic Letters* **78 (1)**: 79–86.

Feenstra, R. C. and Weinstein, D. E. (2017). Globalization, markups, and us welfare, *Journal of Political Economy* **125**: 1040–74.

Holt, M. T. and Goodwin, B. K. (2009). The almost ideal and translog demand systems, *Munich Personal RePEc Archive* **MPRA Paper No. 15092**.