A SECOND COMPARISON OF PARTIAL EQUILIBRIUM MODELS
OF TRQS WITH SENSITIVITY ANALYSIS

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A Second Comparison of Partial Equilibrium Models of TRQs with Sensitivity Analysis

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Abstract

In this short paper, we derive a form of the standard endogenous price Armington CES trade model to estimate the effect of a tariff rate-quota (TRQ) on industry specific imports. We then detail how to model changes to the in-quota and out-of-quota tariff rates. We additionally present a refined derivation of a Dixit-Stiglitz-Krugman (DSK) CES style model of trade and discuss its use in modeling the effects of implementing a TRQ. Finally, we estimate the effects of introducing a TRQ and compare the estimates of the endogenous price Armington CES model to the exogenous price Armington model and the Dixit-Stiglitz-Krugman (DSK) CES model of trade. We show that for the Armington models, only changes to the effective marginal rate affect prices and quantities. We find that the exogenous price Armington model and DSK model produce identical predictions of the impact of the TRQ on subject import prices and comparable predictions about changes in imported volumes and domestic production. The endogenous price Armington model allows for partial pass-through of the TRQ to the subject import price. Consequently, it predicts more modest effects on consumer prices and quantities than the other two models.

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1. Introduction

A tariff rate quota (TRQ) is a tariff schedule with a step: there is an in-quota tariff rate on import volumes below the quota volume and a higher out-of-quota tariff rate on imports above the quota volume. In this paper, we model the impact of a TRQ on the volumes of subject imports and domestic production using an endogenous price Armington CES model with perfect competition. In the Armington model, all of the adjustments happen on the intensive margin of trade. Consequently, only changes to the effective marginal tariff rate impact prices and quantities. The effective marginal rate is the rate paid on the last or marginal unit imported. This is the in-quota rate when the TRQ does not fill and the out-of-quota rate when the TRQ fills. Therefore, modeling a TRQ is equivalent to modeling a change in the import tariff rate.

This paper extends the work of Hallren and Riker (2017) by allowing for endogenous prices in the Armington CES model. In their previous work, Hallren and Riker (2017) compare the outcomes resulting from a TRQ that are predicted by an exogenous price Armington CES model with perfectly elastic import supply and a Dixit-Stiglitz-Krugman (DSK) CES model with fixed costs of trade and production. In the exogenous price Armington CES model, all adjustments occur on the intensive margin of trade and most of the adjustments are reflected in changes to quantity demanded; only the delivered import price is affected by the TRQ. This model potentially under-predicts the effect of imposing a TRQ on domestic prices. In contrast, in a Dixit-Stiglitz-Krugman CES model, both the in-quota and out-of-quota rate affect prices and quantities.

We further extend this previous work by describing the decision rules used to solve industry-specific partial equilibrium models with TRQs. Additionally, we illustrate how TRQs can generate TRQ rents for importing firms if the TRQ fills.

The paper proceeds as follows: In section 2, we derive the endogenous price Armington CES model, and we describe the decision rules used to determine if the TRQ fills. We illustrate the TRQs in the Armington models in section 3. We give comparable details on the setup, equilibrium solution methodology, and TRQ dynamics for a DSK style model in section 4. In section 5, we present a comparative simulation exercise. We demonstrate how to incorporate sensitivity analysis by sampling parameter values and present an example case in section 6. We give concluding remarks in section 7.
2. Non-Linear Armington CES Model with Finite Elasticities of Supply and Perfect Competition

We derive the non-linear Armington CES partial equilibrium modeling following the derivations in Armington (1969) and Hallren and Riker (2017). We then incorporate a TRQ in a fashion similar to Fetzer (2008).

The consumer prices for the three varieties of products, including any tariffs, are $p_d$, $p_s$, and $p_n$. The producer price of the domestic product is $p_d$, while the producer prices of the two varieties of imports are equal to $\frac{p_s}{1+\tau_s}$ and $\frac{p_n}{1+\tau_n}$. The trade cost factor $\tau_s$ is equal to the ad valorem equivalent rate of the tariff and international transport costs on subject imports, and $\tau_n$ is equal to the ad valorem equivalent rate of the tariff and international transport costs on non-subject imports.

Subject country imports face a TRQ, where subject imports face a tariff rate of $\tau_s^{in}$ below the quota and $\tau_s^{out}$ above the quota.

The model focuses on a single national market. Consumers in the market can be a combination of households and industrial users, depending on the industry analyzed.

Equations (1) to (5) are supply curves for the three varieties of products in the industry.

(1) $q_d = a_d \left( p_d \right)^{\epsilon_d}$

(2) Below the quota $q_s = a_s \left( \frac{p_s}{1+\tau_s^{in}} \right)^{\epsilon_s}$

(3) At the quota $q_s = \bar{q}_s$

(4) Above the quota $q_s = a_s \left( \frac{p_s}{1+\tau_s^{out}} \right)^{\epsilon_s}$

(5) $q_n = a_n \left( \frac{p_n}{1+\tau_n} \right)^{\epsilon_n}$

The parameters $\epsilon_d$, $\epsilon_s$, $\epsilon_n$ are constant price elasticities of supply, and $a_d$, $a_s$, $a_n$ represent factors that shift the supply curves. The equations for the supply curves assume a specific form (in this case, they are log-linear), and they are tailored to the industry by fitting the supply shift parameters to industry data. The calibrated values of the supply shifters reflect a variety of factors, including
the level of production capacity and input costs. The model assumes that there is perfect
competition in product markets but that each national variety is an imperfect substitute for the
others. With the TRQ in place, the supply curve for the subject variety becomes a piecewise function
as illustrated in figure 1. Equation (2) is the supply curve for the subject variety below the TRQ,
equation (3) is the supply curve at the quota, and equation (4) is the supply curve above the quota
when subject variety imports face the out-of-quota rate, \( \tau_s^{out} \).

Equation (6) represents total demand in the industry, \( Q \).

\[
(6) \quad Q = k_A P^\theta
\]

The variable \( P \) is a price index for the products of the industry in the national product market, and
the variable \( Y \) represents aggregate expenditure on the product if \( P = 1 \). Equations (7), (8), and (9)
are Constant Elasticity of Substitution (CES) demand curves for the three varieties of products.

\[
(7) \quad q_d = Q b_d^\sigma \left( \frac{p_d}{P} \right)^{-\sigma}
\]

\[
(8) \quad q_s = Q b_s^\sigma \left( \frac{p_s}{P} \right)^{-\sigma}
\]

\[
(9) \quad q_n = Q b_n^\sigma \left( \frac{p_n}{P} \right)^{-\sigma}
\]

The parameter \( \theta \) is the price elasticity of total demand in the industry. The parameters \( b_d, b_s, \) and
\( b_n \) represent factors that shift the demand curves. The equations for the demand curves also
assume specific functional forms (in this case, they are log linear in prices and the price index, and
the price index has a CES functional form). These equations are also tailored to the industry by
fitting the demand shift parameters to industry data. The calibrated values of the demand shifters
reflect a variety of factors, including prices in other industries.

Fetzer (2005) and Hallren and Riker (2017) describe the calibration process wherein the
parameters \( a_i, b_i^\sigma, \) and \( k_A \) are defined and their values are set such that the model correctly predicts
the initial equilibrium -- before the TRQ -- market shares when all prices are set to one. \( k_A \) is the
initial industry expenditure, the \( a_i \) terms are the initial equilibrium quantities supplied, and the
parameters \( b_i^\sigma \) are the initial market shares of each variety \( i \). For each variety, we set quantity
supplied equal to quantity demanded, and the parameters \( a_i, b_i^\sigma, \) and \( k_A \) cancel. Therefore, the
calibration process generates the system of non-linear equations below. To solve this non-linear
version of the model, we use an iterative algorithm to find the set of prices with the TRQ in place that ensures that quantity supplied equals quantity demanded in all markets simultaneously, or equivalently that simultaneously satisfies equations (10) to (14).

\[(10) \quad p^e_d = \frac{p^{\sigma+\theta}}{p^d}\]

\[(11) \quad \text{Below the quota} \left(\frac{p_s}{1+\tau_{in}}\right)^{\varepsilon_s} = \frac{p^{\sigma+\theta}}{p^s}\]

\[(12) \quad \text{At the quota} \left(\frac{p_s}{1+\tau_{in}}\right)^{\varepsilon_s} = q_s\]

\[(13) \quad \text{Above the quota} \left(\frac{p_s}{1+\tau_{out}}\right)^{\varepsilon_s} = \frac{p^{\sigma+\theta}}{p^s}\]

\[(14) \quad \left(\frac{p_n}{1+\tau_n}\right)^{\varepsilon_n} = \frac{p^{\sigma+\theta}}{p^n}\]

Because the supply function for the subject variety is only continuous over specific intervals we have to be more creative when trying to solve the system of equations. Instead of solving a model with three equations, we wrote a program that solves the three models sequentially – each a standard Armington CES model – and then choose the final set of results based on a decision rule used for determining the equilibrium outcome.

Our first model is equations (10), (11), and (14): the domestic market, the subject imports at the below quota rate, and the non-subject imports. We solve the set of consumer prices that allow all three markets to clear simultaneously. If the market clearing quantity of subject imports, \(q_s\), is less than the quota, then we take these results as our prevailing equilibrium because the quota is non-binding. If the market clearing quantity is above the quota, then we numerically solve the second model, equations (10), (13), and (14). Again this is the standard Armington model, but now subject imports face the higher, above quota rate. If the market clearing quantity in the subject market, \(q_{s2}\), is above the quota, then we take the results from this model and conclude that the quota is binding and demand for subject imports is sufficient to consume these imports at out-of-quota quantities despite the higher rate. If \(q_{s2}\) is below the quota when buyers face the higher tariff rate, we conclude that the TRQ is just binding and market demand for subject imports is satisfied at the quota. The diagram below illustrates the decision rule for solving the industry model.
\[
q_s: \begin{cases} 
q_s = q_s^{in} & \text{if } q_s(\tau_s) < q \\
q_s = q_s^{out} & \text{if } q_s(\tau_s) > q_s^{out}
\end{cases}
\]

3. Graphical Illustration of Armington Models with TRQs

Figure 2 shows three possible cases resulting from imposing a TRQ on imports from subject countries. The curve \(S\) is the inverse supply curve for subject imports, with respect to consumer’s market price, without a TRQ, and the stepwise curve \(S_{TRQ}\) is the new inverse supply curve after the TRQ comes into effect. The top panel illustrates the equilibrium when demand is satisfied at an output below the quota. Before the TRQ, the equilibrium quantity is above the quota but falls below the quota when consumers face the in-quota rate \((\tau_s)\). The increase in the marginal tariff rate causes quantity demanded to fall to \(q^*\), the equilibrium market price for subject imports rises to \(p_s^*\), and the government collects tariff revenues (TR) equal to \(\frac{\tau_s}{1 + \frac{\tau_s}{1 + \tau_s}} p_s^* q_s^*\). This is the shaded area in the graph.

The middle panel shows the case where demand for subject imports is such that demand is satisfied at the quota amount after the TRQ is imposed. Because of the difference in marginal tariff rates above and below the tariff, the supply curve \(S_{TRQ}\) has a perfectly inelastic region with a height of \((\tau_s^{out} - \tau_s^{in}) (\frac{q}{a_s})^{1/\epsilon_s}\). Consequently, once demand hits the quota, buyers will, for this range of prices, only be able to consume a fixed amount of subject imports. This generates the TRQ rent. The TRQ rent is the increase in price enjoyed by importing firms that is due, not to the effective marginal rate, but to this perfectly inelastic portion of the supply curve. The amount of the rent per unit is

\[
\left(\frac{p_s^*}{1 + \tau_s^{in}}\right) - (1 + \tau_s^{in}) \left(\frac{\bar{q}}{a_s}\right)^{1/\epsilon_s}.
\]

The remaining gap between the market price and the upper bound of the first section of inverse supply curve \((1 + \tau_s^{in}) \left(\frac{\bar{q}}{a_s}\right)^{1/\epsilon_s}\) is due to the in-quota tariff. The revenue collected by the government is area \(a\) in the graph and is equal to \(\frac{\tau_s^{in}}{1 + \tau_s^{in}} p_s^* \bar{q}_s\).

The bottom panel shows the scenario where the market clears at a quantity above the quota, and the marginal importer faces the out-of-quota tariff. In this scenario, in the Armington model, changes to the in-quota tariff rate will not affect the equilibrium outcome because tariff savings
from a reduction in the in-quota rate are not passed to out-of-quota importers. Here importers face the price, $p_s^*$, and the government collects revenue on the out-of-quota portion and the in-quota portion of imports, all valued at the market price. Therefore, government revenue are the shaded areas $a$ and $b$ in the group and are equal to equal to 

\[
\left( \frac{\tau_{in}}{1+\tau_{in}} \right) p_s^* q_s + \left( \frac{\tau_{out}}{1+\tau_{out}} \right) p_s^* (q_s^* - q_s).
\]

Figure 3 presents these same three cases but for the Armington CES model with perfectly elastic supply (i.e. exogenous prices). The three cases are qualitatively the same, though more simply illustrated.

4. Modeling TRQs with a DSK style model

The Armington (1969) model of trade is a perfect competition, imperfect substitutes model of international trade. The Dixit-Stiglitz-Krugman (DSK) style model of trade allows for imperfect substitutes and monopolistic competition. Hallren & Riker (2017) introduces an industry specific, discrete product space DSK model, based on Dixit and Stiglitz (1977) and Krugman (1980). This section provides some clarifying details.

In the DSK model, consumers maximize a CES utility function. The set of product varieties in the model is the set of firms, each with its own unique variety. In this basic case, there are 3 categories of firms: domestic firms, firms in countries subject to the policy change, and firms in the rest of the world (ROW) in non-subject countries. Within each of the categories, the firms have the same origin and cost structure, and their products are symmetrically differentiated. In each country market, firms engage in monopolistic competition. Firms face a source country specific and constant marginal cost and source country specific fixed cost.

Equation (16) represents total demand in the industry, $Q$.

\[
Q = k_A p^\theta
\]

The CES utility function assumption generates the demand function for each source country variety $j$: \(^1\)

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\(^1\) See Melitz (2003) for derivation of this demand curve.
\[ q_j = b_j^\sigma k_A P^{\sigma+\theta} \left( pp_j (1 + \tau_j) \right)^{-\sigma} \]

The term \( pp_j \) is the producer’s price for variety \( j \). Firms face constant, source country specific marginal costs, and the producer’s price is the constant mark-up over marginal costs. This constant markup embeds the monopolistic competition assumption that there is a continuum of firms, and each firm is “atomistic” (but has monopoly power in its variety). Each firm takes the CPI as given when setting its price. Each firm perceives that it faces a price elasticity of demand of minus sigma, though that’s only true in the limit.

\[ pp_j = \left( \frac{\sigma}{\sigma-1} \right) c_j \]

Given this, the consumer’s price for each variety \( j \) is

\[ p_j = pp_j (1 + \tau_j) \]

And the resulting CPI is

\[ P = \left( \sum n_j b_j^\sigma (p_j)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]

Demand for each firm producing variety \( j \) is:

\[ \bar{q}_j = \left( \frac{1}{n_j} \right) b_j^\sigma k_A P^{\sigma+\theta} \left( pp_j (1 + \tau_j) \right)^{-\sigma} \]

Here \( n_j \) is the number of firms operating in the market in country \( j \), \( b_j^\sigma \) is calibrated to the initial market share divided by the number of firms in market \( j \) in the initial equilibrium (\( b_j^\sigma = \frac{m_{j0}}{n_{j0}} \)), \( k_A \) is the initial industry expenditure, \( P \) is the consumer price index (CPI), \( \sigma \) is the Armington constant elasticity of substitution across source country varieties, and \( \tau_{j0} \) is the initial, effective marginal tariff rate, which is most applications is zero at the initial equilibrium.

It follows that the firm’s profit function from source country \( j \) without a TRQ, facing producer price (\( pp_j \)), and source country specific fixed costs (\( f_j \)) is

\[ \pi_j = (pp_j) \bar{q}_j - c_j \bar{q}_j - f_j \]

Using simple algebra, we show that a firm’s profit function in the initial equilibrium is
\[
\pi_{j0} = \left(\frac{1}{n_{j0}\sigma}\right) b_j^\sigma k_A p^{\sigma+\theta} \left(pp_j (1 + \tau_{j0})\right)^{-\sigma} pp_j - f_j
\]

Here \(\tau_{j0}\) is the initial, effective marginal tax rate on variety \(j\) products and \(n_{j0}\) is the number of firms producing variety \(j\) in the initial equilibrium. In equation (20), firms face the market price, \(pp_j(1 + \tau_{j0})\), in the demand equation. However, firms only collect the producer price, \(pp_j\). In equilibrium with free entry and exit, all firms earn zero economic profit. Using this zero profit condition (ZPC), fixed costs equal

\[
f_j = \left(\frac{1}{n_{j0}\sigma}\right) b_j^\sigma k_A p^{\sigma+\theta} \left(pp_j (1 + \tau_{j0})\right)^{-\sigma} pp_j
\]

Under a binding TRQ, the firms will pay the in-quota rate \((\tau_{j1}^{in})\) on some units and the out-of-quota rate \((\tau_{j1}^{out})\) on other units. The model needs to account for the difference in total tariff revenue paid under a TRQ versus a uniform ad-valorem rate. We call this term tariff savings (TS). Figure 4 illustrates the difference in tariff rates paid on subject imports between the standard ad-valorem tariff case and the out-of-quota TRQ case.

\[
TS_j = TR_1 - TR_2 - TR_3
\]

\[
TS_j = \left(\frac{\tau_{j1}^{out}}{1+\tau_{j1}^{out}}\right) - \left(\frac{\tau_{j1}^{in}}{1+\tau_{j1}^{in}}\right) p_j quota_j
\]

If we divide equation (9) by the number of firms, \(n_j\), then we get the tariff savings per-firm \((ts_j)\). The resulting profit function is

\[
\pi_j = \left(\frac{1}{n_j\sigma}\right) b_j^\sigma k_A p^{\sigma+\theta} \left(pp_j (1 + \tau_j)\right)^{-\sigma} pp_j - f_j + ts_j
\]

\(\tau_j\) is the final, effective marginal rate.

Our algorithm uses the zero profit condition to determine the equilibrium number of firms producing each variety \((n_d, n_s, n_n)\) in the new equilibrium after the policy shock. There is one exception, however. When a TRQ is binding at the quota (i.e. when the demand curve passes through the vertical portion of the marginal cost curve of the subject variety), the algorithm needs to take into account both the zero profit condition to generate the optimal number of firms and the market clearing condition to predict the market clearing price for the subject variety.
5. Example Simulation

We run four identical experiments on each of our three models: exogenous price Armington, endogenous price Armington, and DSK. To maximize comparability between the DSK and Armington models, we set number of varieties \( n \) in each country in the DSK equal to 1, as in Hallren and Riker (2017). We design our experiments such that each model has the same qualitative outcome so as to highlight the differences in predicted changes to quantities and prices across models.

Table 1 lists each of the experiments and the qualitative outcome. In scenario 1, we impose a standard, no quota, ad-valorem tariff of 10%. In scenario 2, we impose a TRQ within an in-quota rate of 0%, an out-of-quota rate at 10%, and a quota of 15. In scenario 2, the TRQ is binding: the effective marginal rate is 10%. For the Armington models the results should be identical between scenario 1 and scenario 2. However, because firms in the DSK model capture tariff savings when the in-quota rate is lower than the out-of-quota rate the predicted results between the two scenarios will not be the same. In scenario 3, we set the in-quota rate to 10%, the out-of-quota rate at 50%, and the quota at 15. In this scenario, the TRQ is binding at the quota. In the last experiment, the TRQ is non-binding: the effective marginal rate is the in-quota rate. In this case, the in-quota rate is 40%, the out-of-quota rate is 50%, and the quota is 15.

We simulate the effects of the policies on the volumes of subject imports and domestic production using the specific model inputs listed in Table 2. To illustrate the differences among the policy alternatives, we make several assumptions about market shares and elasticities. We assume that domestic producers have a 60 percent market share, while subject imports have a 30 percent share. We assume that the total size of the market is 100 units, while the quota volume is 15 units. We assume that the elasticity of substitution among varieties from different sources is 4. Additionally, we assume the supply elasticities in the non-linear Armington model are 1 for domestic firms and 10 for foreign firms. Finally, the aggregate price elasticity of demand is -1.

Table 3 presents the results of experiment 1. Here we impose a 10% ad-valorem tariff. The predicted price effects are almost identical between the exogenous price Armington and DSK models. The price on subject imports increases by the amount of the tariff and all other prices remain unchanged. The market price of subject imports does not increase by the full amount of the tariff in the endogenous price Armington case. Here the increase in the price of subject imports

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2 These assumptions should be replaced with actual market data when the model is applied.
causes demand to increase for all other varieties and this shift raises the market price of all other
varieties. Thus in contrast to the other two models, the endogenous price Armington allows tariffs
on one variety to have spillover effects on the prices of other varieties. Consequently, the
endogenous price Armington also consistently predicts a larger increase in the CPI from a given
tariff shock than either of the other two models.

On the quantity side too, the predicted changes between the exogenous price Armington and the
DSK model are nearly identical. The only difference is in the volume of subject imports, the DSK
predicts a lower quantity of subject imports. Relative to these two models, the endogenous price
Armington predicts smaller changes in domestic quantities and subject imports.

In scenario 2, we impose a TRQ with an in-quota rate of 0% and an outside rate of 10%. Here the
TRQ is binding so the scenario is identical to the first experiment, except the in-quota rate is now
0%. The results for both prices and quantities for the Armington models are the same as in scenario
1 (see table 4). However, because firms in the DSK model can capture savings when the in-quota
rate is lower than the out-of-quota rate, the DKS results are different. The market price of subject
imports still increases by 10% but the CPI rises by only 1.6% instead of 2.7%, as in experiment 1.
The quantity of subject imports falls by more than in scenario 1 but the number of firms only falls
by 1.6%, where as in scenario 1 the number of firms falls by 26.1%. Under a straight tariff, the
decline in subject imports occurs, in almost equal measure, along both the intensive and extensive
margins. Under a binding TRQ, the effect occurs primarily along the intensive, rather than extensive
margin, because of the tariff savings that firms are able to capture.

In scenario 3, we re-impose a 10% in-quota rate and increase the out-of-quota rate to 50%. This
ensures that the TRQ is binding at the quota. Table 5 shows the results of this experiment. In the
case of the Armington models, when a TRQ is binding at the quota, the market price will increase by
more than the in-quota rate but by less than the out-of-quota rate. In contrast to all other
experiments, the price of subject imports increases by more in the endogenous price model
(28.5%) than in the exogenous price Armington model (23.5%). Additionally, the price of the
domestic variety increases by 6.4% in the endogenous price model, resulting in a total change in the
CPI of 10.9%, double the change of the CPI in the exogenous price Armington.

The volume of subject imports in all three models is 15 units. The percent change in subject imports
is identical across both Armington models, but domestic production increases by more in the
exogenous price Armington model (16.4%) than in the endogenous price model (6.4%).
In contrast to all other scenarios, the DSK model predicts the smallest increase in subject import prices (13.3%). The model further predicts that subject imports fall by 33.3% and domestic output increases by 9.8%.

In the last experiment (table 6), we increase the in-quota rate to 40% and keep the out-of-quota rate at 50%. The high in-quota rate ensures that the TRQ is not binding for any of the three models. Qualitatively, this scenario is similar to the straight tariff case (experiment 1). The price of subject imports increases by the full amount of the effective marginal tariff (40%) for both the exogenous price Armington model and the DSK model. The all other varieties’ prices remain unchanged. Between the two models, the exogenous price Armington predicts a larger increase in the CPI (7.3%) than the DSK model does (5.4%). The endogenous Armington predicts a smaller increase in the price of subject imports (30.1%) than the other two models but allows for a spillover price increase on the domestic variety of 6.7%. Consequently, this model generates the largest CPI increase of 11.4%.

On the quantity side, of the three models, the endogenous price Armington model predicts the smallest changes, in absolute value, in domestic output (6.7%) and subject imports (-51.8%). The DSK model predicts the largest decline in subject imports (-69.5%), and the exogenous price Armington predicts the biggest increase in domestic production.

In our experiments, except for the case where TRQ binds at the quota, the following patterns appear. The exogenous Armington and DSK models generate very similar changes to subject import prices and no changes to domestic or non-subject import prices. The change in subject import prices in these models is larger than in the endogenous price Armington model. The endogenous price Armington model allows for a spillover effect of the TRQ onto domestic and non-subject import prices. Consequently, the endogenous price Armington model predicts the largest increase in the overall price level resulting from a TRQ.

On the quantity side, the DSK model consistently predicts the largest decline in the volume of subject imports. Again this is because the TRQ reduces subject imports on both the intensive and extensive margins. Of the three models, the ex-post volume of subject imports is the lowest across all three simulations. With respect to changes in domestic production, the exogenous price Armington model predicts the largest increase in domestic output.

Between the two versions of the Armington model, the volume of subject imports falls by more in the exogenous price version because it predicts a larger increase in the relative price of subject
imports to domestic goods. In contrast to the other two models, the endogenous price Armington model predicts a modest increase in domestic production, only about one-fifth as large as that predicted by DSK. This is because the endogenous price model predicts the smallest relative change in prices between subject and domestic goods. Additionally, it generates the largest change in the overall price level.

6. Sensitivity Analysis

In these models of trade there are five behavioral parameters: a price elasticity of supply for each of the three varieties, an overall industry price elasticity of demand, and an Armington elasticity. In some cases, we may have econometric estimates of some or all of these parameters for the industry of interest. In these cases, we can use Monte Carlo simulation (for example Hallren and Opanasets (2017)) to incorporate parameter uncertainty and generate standard errors around our predicted changes to prices and quantities in each policy experiment.

In cases where we do not have econometric estimates, we can use qualitative information to establish upper and lower bound values for each of the behavioral parameters. We run the model with each possible combination of parameter values, \( N^2 \) cases. Here \( N \) is the number of parameters. This method generates an upper and lower bound estimate for each outcome of interest.

To demonstrate this method, we repeat experiment 2 using the range of parameter values in table 7. We present the results in table 8. In this experiment, whether the TRQ is binding exactly at the quota or above the quota is sensitive to the selection of parameter values. Therefore, in some cases the effective marginal rate is 0% and in others is 10%. Consequently, the price results are non-robust to adjustments of supply and demand elasticities. The effect of the TRQ on subject import quantities and number of firms producing the subject varieties has the anticipated sign, though the magnitude of the effect has a wide range. The effect of the TRQ on subject import quantities has a range of about 5 percentage points, and the estimated effect of the TRQ on the number of subject firms has a range of 12 percentage points. Interestingly, in some cases the TRQ increases the overall number of firms by as much as 2.2%, though in all cases real industry output declines.

This example shows how both the qualitative results (e.g. if or where the TRQ is binding) and qualitative results are often sensitive to parameter values. Therefore, it is important to incorporate parameter uncertainty into the analysis by conducting and reporting sensitivity analysis.
7. Conclusions and Areas for Future Research

We assess the effects of a TRQ on imports and domestic production using three different types of partial equilibrium models, an exogenous price Armington CES model, an endogenous price Armington CES mode, and a Dixit-Stiglitz-Krugman (DSK) CES model of trade, and compare their performance. In the Armington model with only adjustment on the intensive margin of trade, a TRQ that fills has the same effect on trade as a flat tariff at the out-of-quota rate. In the DSK model, on the other hand, the two policies are not equivalent and the in-quota rate has an effect on trade and domestic production even when the TRQ fills.

Moving forward, the next steps are to explicitly incorporate vertically integrated, multi-tiered supply chains for the three models. Once these are in place, it becomes possible to model adjustments over distinct time horizons: short-run, medium-run, and long-run. Following these additions, then it is feasible to considering designing these industry specific models as recursive, dynamic industry specific models.
Figure 1. TRQ in an Endogenous Price Armington Model

\[ q_s > \bar{q}: q_s = a_s \left( \frac{p_s}{1 + \tau_{s_2}} \right)^{\varepsilon_s} \]

\[ q_s = \bar{q}: q_s = \bar{q} \]

\[ q_s \leq \bar{q}: q_s = a_s \left( \frac{p_s}{1 + \tau_{s_1}} \right)^{\varepsilon_s} \]
Figure 2. Equilibrium outcomes with a TRQ and endogenous prices

$P$ vs. $Q$

- $S_{TRQ}$
- $S$
- $D$
- $q^*_s$
- $\bar{q}_s$
- $\frac{p_s^*}{(1 + \tau_s^{lin})}$
- $\frac{p_s^*}{(1 + \tau_s^{out})}$
- $TRQ \text{ rent}$
- $a$
- $b$
- $\alpha$

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Figure 3. Equilibrium outcomes with a TRQ and exogenous prices
Figure 4. Tariff savings under TRQ vs standard Ad-Valorem on subject imports in DSK framework
Table 1. Policy Scenarios

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quota</strong></td>
<td>.</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td><strong>In-Quota Rate</strong></td>
<td>10%</td>
<td>0%</td>
<td>10%</td>
<td>40%</td>
</tr>
<tr>
<td><strong>Out-of-Quota rate</strong></td>
<td>10%</td>
<td>10%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td><strong>Scenario Description</strong></td>
<td>Standard AVE at 10%</td>
<td>Binding TRQ at 10%</td>
<td>TRQ binding at the quota</td>
<td>Non-binding TRQ</td>
</tr>
</tbody>
</table>
### Table 2. Model Inputs

<table>
<thead>
<tr>
<th>Input</th>
<th>Region</th>
<th>Domestic</th>
<th>Subject</th>
<th>Non-Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Elasticity</td>
<td></td>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Market Share</td>
<td></td>
<td>60%</td>
<td>30%</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Global Inputs**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Armington Elasticity</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Price Elasticity of Total Demand</td>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>Total Market Size</td>
<td></td>
<td></td>
<td>$100.00</td>
</tr>
</tbody>
</table>
Table 3. Results of Scenario 1
(Inside rate (10%), Outside rate (10%), Quota (\text{..}))

<table>
<thead>
<tr>
<th></th>
<th>Exogenous Price Armington</th>
<th>Endogenous Price Armington</th>
<th>Dixit-Stiglitz-Krugman (DSK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pct Chg in Price of Dom Production</td>
<td>0.0%</td>
<td>2.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pct Chg in Price of Subj Imports</td>
<td>10.0%</td>
<td>7.9%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Pct Chg in Overall Prices</td>
<td>2.6%</td>
<td>3.6%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Quantity Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of subject imports</td>
<td>22.1</td>
<td>24.6</td>
<td>16.6</td>
</tr>
<tr>
<td>Pct Chg in Domestic Production</td>
<td>8.1%</td>
<td>2.1%</td>
<td>8.2%</td>
</tr>
<tr>
<td>Pct Chg in volume of Subj Imports</td>
<td>-26.2%</td>
<td>-17.9%</td>
<td>-26.1%</td>
</tr>
</tbody>
</table>
### Table 4. Results of Scenario 2
(Inside rate (0%), Outside rate (10%), Quota (15))

<table>
<thead>
<tr>
<th></th>
<th>Exogenous Price</th>
<th>Endogenous Price</th>
<th>Dixit-Stiglitz-Krugman (DSK)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Armington</td>
<td>Armington</td>
<td></td>
</tr>
<tr>
<td>Price Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pct Chg in Price of Dom Production</td>
<td>0.0%</td>
<td>2.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pct Chg in Price of Subj Imports</td>
<td>10.0%</td>
<td>7.9%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Pct Chg in Overall Prices</td>
<td>2.6%</td>
<td>3.6%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Quantity Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of subject imports</td>
<td>22.1</td>
<td>24.6</td>
<td>16.1</td>
</tr>
<tr>
<td>Pct Chg in Domestic Production</td>
<td>8.1%</td>
<td>2.1%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Pct Chg in volume of Subj Imports</td>
<td>-26.2%</td>
<td>-17.9%</td>
<td>-28.5%</td>
</tr>
</tbody>
</table>
Table 5. Results of Scenario 3
(Inside rate (10%), Outside rate (50%), Quota (15))

<table>
<thead>
<tr>
<th></th>
<th>Exogenous Price Armington</th>
<th>Endogenous Price Armington</th>
<th>Dixit-Stiglitz-Krugman (DSK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pct Chg in Price of Dom Production</td>
<td>0.0%</td>
<td>6.4%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pct Chg in Price of Subj Imports</td>
<td>23.5%</td>
<td>28.5%</td>
<td>13.3%</td>
</tr>
<tr>
<td>Pct Chg in Overall Prices</td>
<td>5.2%</td>
<td>10.9%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Quantity Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of subject imports</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Pct Chg in Domestic Production</td>
<td>16.4%</td>
<td>6.4%</td>
<td>9.8%</td>
</tr>
<tr>
<td>Pct Chg in volume of Subj Imports</td>
<td>-50.0%</td>
<td>-50.0%</td>
<td>-33.3%</td>
</tr>
</tbody>
</table>
Table 6. Results of Scenario 4
(Inside rate (40%), Outside rate (50%), Quota (15))

<table>
<thead>
<tr>
<th>Price Effects</th>
<th>Exogenous Price Armington</th>
<th>Endogenous Price Armington</th>
<th>Dixit-Stiglitz-Krugman (DSK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pct Chg in Price of Dom Production</td>
<td>0.0%</td>
<td>6.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pct Chg in Price of Subj Imports</td>
<td>40.0%</td>
<td>30.1%</td>
<td>40.0%</td>
</tr>
<tr>
<td>Pct Chg in Overall Prices</td>
<td>7.3%</td>
<td>11.4%</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity Effects</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of subject imports</td>
<td>9.6</td>
<td>14.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Pct Chg in Domestic Production</td>
<td>23.6%</td>
<td>6.7%</td>
<td>17.2%</td>
</tr>
<tr>
<td>Pct Chg in volume of Subj Imports</td>
<td>-67.8%</td>
<td>-51.8%</td>
<td>-69.5%</td>
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</tbody>
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Table 7. Parameters for Scenario 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Domestic</th>
<th>Subject</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Elasticity</td>
<td>[1.0, 1.5]</td>
<td>[5.0, 15.0]</td>
<td>[5.0, 15.0]</td>
</tr>
</tbody>
</table>

**Global Inputs**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armington Elasticity</td>
<td>4</td>
</tr>
<tr>
<td>Price Elasticity of Total Demand</td>
<td>[-0.5, -1.5]</td>
</tr>
</tbody>
</table>
Table 8. Results of Scenario 5
(Inside rate (0%), Outside rate (10%), Quota (15))

<table>
<thead>
<tr>
<th></th>
<th>Domestic</th>
<th>Subject</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Effects</td>
<td>[0.0%, 0.0%]</td>
<td>[8.2%, 10%]</td>
<td>[1.6%, 2.2%]</td>
</tr>
<tr>
<td>Quantity Effects</td>
<td>[5.6%, 5.6%]</td>
<td>[-27.9%, -23.0%]</td>
<td>[-4.4%, -3.0%]</td>
</tr>
<tr>
<td>Firm Effects</td>
<td>[5.6%, 5.6%]</td>
<td>[-16.7%, -4.8%]</td>
<td>[-1.8%, 2.2%]</td>
</tr>
<tr>
<td>Marginal Tariff Rate</td>
<td>[0%, 10%]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tariff Revenue</td>
<td>[$0.00, $0.50]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
References


