Three Regions Tariff Model

D. Riker, 05/13/19 version

This partial equilibrium (PE) model of ad valorem tariffs has three sources of supply and three regional markets, labeled A, B, and C. Consumer demands have a non-nested CES form for the products of the industry. Total industry demand in the region has a constant price elasticity that is region-specific. The supply for each of the three sources has a constant price elasticity. The data inputs of the model are the initial expenditures, initial tariffs, and revised tariffs. The parameter inputs of the model are the elasticity of substitution, the price elasticity of total demand, and the price elasticity of each of the sources of supply. The model also includes demand and supply shift parameters that are calibrated to initial market equilibrium prices and quantities.

The model simulates the effects on prices, quantities, and consumer expenditures of a change in ad valorem tariff rates.

The user can modify data inputs, elasticity values, and tariff rates in the simulation by change the values in the ORANGE-shaded lines in the notebook below tab. The spreadsheet will update the estimated changes in economic outcomes that are reported in the GREEN-shaded cells once the user selects “Evaluate Notebook” under “Evaluation” in the Menu above.

This model is provided as a generic analytical tool, and the data and parameter values are fictional and illustrative. Actual data and parameter values should be supplied by the user based on the industry and market to which the model is applied. The model is the result of ongoing professional research of USITC staff and may be updated. The model is not meant to represent in any way the view of the U.S. International Trade Commission or any of its individual Commissioners. The model is posted to promote the active exchange of ideas between USITC staff and experts outside the USITC and to provide useful economic modeling tools to the public.

\[ \text{ClearAll}[f]; \]

**Parameter Inputs**

**Elasticity of Substitution**

\[ \sigma = 3; \]

**Total Industry Price Elasticity of Demand**
<table>
<thead>
<tr>
<th>Region</th>
<th>Supply Elasticity Parameter</th>
<th>Initial Ad Valorem Tariff</th>
<th>Revised Ad Valorem Tariff</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\eta = -1$</td>
<td>$t_{BA0} = 0.10$</td>
<td>$t_{BA} = 0.00$</td>
</tr>
<tr>
<td>B</td>
<td>$\eta = -1$</td>
<td>$t_{CA0} = 0.00$</td>
<td>$t_{CA} = 0.00$</td>
</tr>
<tr>
<td>C</td>
<td>$\eta = -1$</td>
<td>$t_{AB0} = 0.00$</td>
<td>$t_{AB} = 0.00$</td>
</tr>
</tbody>
</table>

Supply Elasticity Parameters

- $e_A = 5$
- $e_B = 5$
- $e_C = 5$

Initial Ad Valorem Tariffs

- $t_{BA0} = 0.10$
- $t_{CA0} = 0.00$
- $t_{AB0} = 0.00$
- $t_{AC0} = 0.00$
- $t_{BC0} = 0.00$
- $t_{CB0} = 0.00$

Revised Ad Valorem Tariff

- $t_{BA} = 0.00$
- $t_{CA} = 0.00$
- $t_{AB} = 0.00$
- $t_{AC} = 0.00$
- $t_{BC} = 0.00$
- $t_{CB} = 0.00$
## Initial Equilibrium Values

### Expenditures in A

| In[ ]= | \( v_{AA0} = 100; \) |
| In[ ]= | \( v_{BA0} = 100; \) |
| In[ ]= | \( v_{CA0} = 100; \) |

### Expenditures in B

| In[ ]= | \( v_{BB0} = 100; \) |
| In[ ]= | \( v_{AB0} = 100; \) |
| In[ ]= | \( v_{CB0} = 100; \) |

### Expenditures in C

| In[ ]= | \( v_{CC0} = 100; \) |
| In[ ]= | \( v_{BC0} = 100; \) |
| In[ ]= | \( v_{AC0} = 100; \) |

### Prices

| In[ ]= | \( p_{A0} = 1; \) |
| In[ ]= | \( p_{B0} = 1; \) |
| In[ ]= | \( p_{C0} = 1; \) |

### Quantities from A

| In[ ]= | \( q_{AA0} = \frac{v_{AA0}}{p_{A0}}; \) |
| In[ ]= | \( q_{BA0} = \frac{v_{BA0}}{p_{B0} (1 + t_{BA0})}; \) |
| In[ ]= | \( q_{CA0} = \frac{v_{CA0}}{p_{C0} (1 + t_{CA0})}; \) |

### Quantities from B
In[1]:= \[ q_{BB0} = \frac{v_{BB0}}{p_{B0}}; \]

In[2]:= \[ q_{AB0} = \frac{v_{AB0}}{p_{A0} \left(1 + t_{AB0}\right)}; \]

In[3]:= \[ q_{CB0} = \frac{v_{CB0}}{p_{C0} \left(1 + t_{CB0}\right)}; \]

Quantities from C

In[4]:= \[ q_{CC0} = \frac{v_{CC0}}{p_{C0}}; \]

In[5]:= \[ q_{BC0} = \frac{v_{BC0}}{p_{B0} \left(1 + t_{BC0}\right)}; \]

In[6]:= \[ q_{AC0} = \frac{v_{AC0}}{p_{A0} \left(1 + t_{AC0}\right)}; \]

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**Calibration of Parameters Based on the Initial Equilibrium**

In[7]:= \[ a_{A} = \left(\frac{v_{AA0}}{p_{A0}} + \frac{v_{AB0}}{p_{A0} \left(1 + t_{AB0}\right)} + \frac{v_{AC0}}{p_{A0} \left(1 + t_{AC0}\right)}\right) p_{A0}^{-e_{A}}; \]

In[8]:= \[ a_{B} = \left(\frac{v_{BB0}}{p_{B0}} + \frac{v_{BA0}}{p_{B0} \left(1 + t_{BA0}\right)} + \frac{v_{BC0}}{p_{B0} \left(1 + t_{BC0}\right)}\right) p_{B0}^{-e_{B}}; \]

In[9]:= \[ a_{C} = \left(\frac{v_{CC0}}{p_{C0}} + \frac{v_{CA0}}{p_{C0} \left(1 + t_{CA0}\right)} + \frac{v_{CB0}}{p_{C0} \left(1 + t_{CB0}\right)}\right) p_{C0}^{-e_{C}}; \]

In[10]:= \[ b_{AB} = \frac{v_{AB0} \left(p_{A0} \left(1 + t_{AB0}\right)\right)^{\text{sigma}-1}}{p_{B0}}; \]

In[11]:= \[ b_{AC} = \frac{v_{AC0} \left(p_{A0} \left(1 + t_{AC0}\right)\right)^{\text{sigma}-1}}{v_{CC0}}; \]

In[12]:= \[ b_{BA} = \frac{v_{BA0} \left(p_{B0} \left(1 + t_{BA0}\right)\right)^{\text{sigma}-1}}{p_{A0}}; \]

In[13]:= \[ b_{BC} = \frac{v_{BC0} \left(p_{B0} \left(1 + t_{BC0}\right)\right)^{\text{sigma}-1}}{p_{C0}}; \]

In[14]:= \[ b_{CA} = \frac{v_{CA0} \left(p_{C0} \left(1 + t_{CA0}\right)\right)^{\text{sigma}-1}}{p_{A0}}; \]

In[15]:= \[ b_{CB} = \frac{v_{CB0} \left(p_{C0} \left(1 + t_{CB0}\right)\right)^{\text{sigma}-1}}{p_{B0}}; \]
New Equilibrium Values with Revised Tariff

\[ PA = \left( pA^{1-\text{sigma}} + bBA \left( pB \left( 1 + tBA \right) \right)^{1-\text{sigma}} + bCA \left( pC \left( 1 + tCA \right) \right)^{1-\text{sigma}} \right)^{\frac{1}{1-\text{sigma}}}; \]

\[ PB = \left( pB^{1-\text{sigma}} + bAB \left( pA \left( 1 + tAB \right) \right)^{1-\text{sigma}} + bCB \left( pC \left( 1 + tCB \right) \right)^{1-\text{sigma}} \right)^{\frac{1}{1-\text{sigma}}}; \]

\[ PC = \left( pC^{1-\text{sigma}} + bBC \left( pB \left( 1 + tBC \right) \right)^{1-\text{sigma}} + bAC \left( pA \left( 1 + tAC \right) \right)^{1-\text{sigma}} \right)^{\frac{1}{1-\text{sigma}}}; \]

\[ kA = vAA \frac{PA^{\text{sigma}-\etaA}}{P\theta^{\text{sigma}-\etaA}} \frac{pA^{\text{sigma}-1}}{P\theta^{\text{sigma}-1}}; \]

\[ kB = vBB \frac{PB^{\text{sigma}-\etaB}}{P\theta^{\text{sigma}-\etaB}} \frac{pB^{\text{sigma}-1}}{P\theta^{\text{sigma}-1}}; \]

\[ kC = vCC \frac{PC^{\text{sigma}-\etaC}}{P\theta^{\text{sigma}-\etaC}} \frac{pC^{\text{sigma}-1}}{P\theta^{\text{sigma}-1}}; \]

\[ \text{FindRoot}[\{\text{EqnA1, EqnB1, EqnC1}, \{pA, pA\theta\}, \{pB, pB\theta\}, \{pC, pC\theta\}] \]

\[ \{pA \rightarrow 0.9977, pB \rightarrow 1.00969, pC \rightarrow 0.9977\} \]
\[q_{BB1} = k_B P_B \sigma B \beta_B \sigma B;\]
\[q_{BA1} = k_A P_A \sigma A \beta_A \sigma A;\]
\[q_{BC1} = k_C P_C \sigma C \beta_C \sigma C;\]
\[q_{CA1} = k_A P_A \sigma A \beta_A \sigma A;\]

**Percent Changes in Producer Prices**

**Supplier A**

\[
\frac{(p_{A1} - p_{A0}) \times 100}{p_{A0}}
\]

Out\(\rightarrow\) = 0.230032

**Supplier B**

\[
\frac{(p_{B1} - p_{B0}) \times 100}{p_{B0}}
\]

Out\(\rightarrow\) = 0.968785

**Supplier C**

\[
\frac{(p_{C1} - p_{C0}) \times 100}{p_{C0}}
\]

Out\(\rightarrow\) = -0.230032

**Percent Changes in Quantities**

**Domestic Shipments in A**

\[
\frac{(q_{AA1} - q_{AA0}) \times 100}{q_{AA0}}
\]

Out\(\rightarrow\) = 5.48559

**Exports of A to B**
\[
\text{In[1]} := \frac{(qAB_1 - qAB_0) \times 100}{qAB_0} \\
\text{Out[1]} = 1.02548
\]

Exports of A to C

\[
\text{In[2]} := \frac{(qAC_1 - qAC_0) \times 100}{qAC_0} \\
\text{Out[2]} = 1.02548
\]

Domestic Shipments in B

\[
\text{In[3]} := \frac{(qBB_1 - qBB_0) \times 100}{qBB_0} \\
\text{Out[3]} = -2.53044
\]

Exports of B to A

\[
\text{In[4]} := \frac{(qBA_1 - qBA_0) \times 100}{qBA_0} \\
\text{Out[4]} = 21.3708
\]

Exports of B to C

\[
\text{In[5]} := \frac{(qBC_1 - qBC_0) \times 100}{qBC_0} \\
\text{Out[5]} = -2.53044
\]

Domestic Shipments in C

\[
\text{In[6]} := \frac{(qCC_1 - qCC_0) \times 100}{qCC_0} \\
\text{Out[6]} = 1.02548
\]

Exports of C to A

\[
\text{In[7]} := \frac{(qCA_1 - qCA_0) \times 100}{qCA_0} \\
\text{Out[7]} = -5.48559
\]

Exports of C to B
Percent Change in Consumer Expenditures on Imports and Domestic Shipments

Domestic Shipments in A

\( \text{Out} = \frac{(pA1 qAA1 - pA0 qAA0) \times 100}{pA0 qAA0} \)

Out = -5.70301

Exports of A to B

\( \text{Out} = \frac{(pA1 (1 + tAB) qAB1 - pA0 (1 + tAB0) qAB0) \times 100}{pA0 (1 + tAB0) qAB0} \)

Out = 0.793084

Exports of A to C

\( \text{Out} = \frac{(pA1 (1 + tAC) qAC1 - pA0 (1 + tAC0) qAC0) \times 100}{pA0 (1 + tAC0) qAC0} \)

Out = 0.793084

Domestic Shipments in B

\( \text{Out} = \frac{(pB1 qBB1 - pB0 qBB0) \times 100}{pB0 qBB0} \)

Out = -1.58617

Exports of B to A

\( \text{Out} = \frac{(pB1 (1 + tBA) qBA1 - pB0 (1 + tBA0) qBA0) \times 100}{pB0 (1 + tBA0) qBA0} \)

Out = 11.406

Exports of B to C

\( \text{Out} = \frac{(pB1 qBC1 - pB0 qBC0) \times 100}{pB0 qBC0} \)

Out = 0.793084
In[1]:= \[
\frac{(pB_1 (1 + tBC) qBC_1 - pB_0 (1 + tBC_0) qBC_0) 100}{pB_0 (1 + tBC_0) qBC_0}
\]

Out[1]= \(-1.58617\)

Domestic Shipments in C

In[2]:= \[
\frac{(pC_1 qCC_1 - pC_0 qCC_0) 100}{pC_0 qCC_0}
\]

Out[2]= \(0.793084\)

Exports of C to A

In[3]:= \[
\frac{(pC_1 (1 + tCA) qCA_1 - pC_0 (1 + tCA_0) qCA_0) 100}{pC_0 (1 + tCA_0) qCA_0}
\]

Out[3]= \(-5.70301\)

Exports of C to B

In[4]:= \[
\frac{(pC_1 (1 + tCB) qCB_1 - pC_0 (1 + tCB_0) qCB_0) 100}{pC_0 (1 + tCB_0) qCB_0}
\]

Out[4]= \(0.793084\)