

STRUCTURAL EQUATIONS FOR PE MODELS  
IN GROUP 2 (IMPERFECT COMPETITION)

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**Abstract**

This note presents the structural equations for the second group of PE simulation models of changes in trade policy. Euler Method spreadsheet versions of these models are available at [https://www.usitc.gov/data/pe\\_modeling/index.htm](https://www.usitc.gov/data/pe_modeling/index.htm).

The models described in this documentation are the result of ongoing professional research of USITC staff and are solely meant to represent the professional research of individual authors. These papers are not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners. Please address correspondence to [david.riker@usitc.gov](mailto:david.riker@usitc.gov).

# 1 Introduction

There are several PE models with imperfect competition that have been translated into a user-friendly spreadsheet format.

## 2 Bertrand Differentiated Products Tariff Model

The first variant of the model relaxes the perfect competition assumption by allowing firms to choose their price to maximize profits. This model is applicable to highly concentrated industries in which imports are an important source of competition and profits are at stake.<sup>1</sup> The Bertrand model assumes non-nested CES demand functions with elasticity of substitution  $\sigma$ . Let  $d$  be the index for the domestic firm,  $s$  be the index for the foreign firm subject to tariffs, and  $n$  be the index for the non-subject foreign firm.  $P$  is the CES price index for the industry,  $p_j$  are the prices of the products of firm  $j \in \{d, s, n\}$ ,  $b_j$  is a preference asymmetry parameter,  $t_s$  is the tariff applied to the subject imports  $q_s$ ,  $c_j$  is the constant marginal cost,  $f_j$  is the fixed cost, and  $\eta$  is the price elasticity of total industry demand. Equations (1) through (4) describe the price index and demand functions.

$$P = \left( (p_d)^{1-\sigma} + b_s (p_s(1+t_s))^{1-\sigma} + b_n (p_n)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (1)$$

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<sup>1</sup>See Riker (2018) for more information on the Bertrand model and simulated results.

$$q_d = k (P)^\eta \left( \frac{p_d}{P} \right)^{-\sigma} \quad (2)$$

$$q_s = k b_s (P)^\eta \left( \frac{p_s(1 + t_s)}{P} \right)^{-\sigma} \quad (3)$$

$$q_n = k b_n (P)^\eta \left( \frac{p_n}{P} \right)^{-\sigma} \quad (4)$$

Profits of firm  $j \in \{d, s, n\}$  are:

$$\pi_j(p_j) = (p_j - c_j) q_j - f_j \quad (5)$$

In Bertrand-style competition, each firm maximizes profits by choosing a price, taking the prices of the other firms as given. Then the first order condition representing the profit-maximizing choice of  $p_j$  is:

$$p_j = (p_j - c_j) \left( \sigma - (\sigma + \eta) \frac{b_j(p_j(1 + t_j))^{1-\sigma}}{\sum_k b_k(p_k(1 + t_k))^{1-\sigma}} \right) \quad (6)$$

### 3 Cournot Tariff Model

The second variant is a Cournot Competition model.<sup>2</sup> This model is also applicable to highly concentrated industries in which imports are an important source of competition but unlike the Bertrand model, this model assumes

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<sup>2</sup>See Riker (2019) for more information on the Cournot model and simulated results.

that foreign and domestic products are perfect substitutes. Suppose there are two firms, a domestic firm ( $d$ ) and a foreign firm subject to tariffs ( $f$ ). Denote  $P$  as the market price,  $\eta$  as the constant price elasticity of demand, and  $\beta$  as a constant calibrated to the size of the market. Then for  $k \in \{d, f\}$ , consumer demand takes on the following functional form:

$$\sum_k q_k = \beta (P)^\eta \quad (7)$$

Profits of firm  $j \in \{d, f\}$  are:

$$\pi_j = (P - c_j(1 + t_j)) q_j - f_j \quad (8)$$

In Cournot-style competition, each firm maximizes profits by choosing a quantity, taking the output of the other firms as given. Then the first order condition representing the profit-maximizing choice of  $q_j$  is:

$$\left( \left( \frac{1}{\beta} \left( \sum_k q_k \right) \right)^{\frac{1}{\eta}} - c_j (1 + t_j) \right) + \left( \frac{1}{\beta} \left( \sum_k q_k \right) \right)^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q_j}{\sum_k q_k} \right) = 0 \quad (9)$$

## 4 Monopoly Tariff Model

The third variant is a domestic monopoly model with a foreign competitive fringe. This model is applicable to industries with a dominant firm who has a large market share and several smaller price-taking firms called fringe firms.

The dominant monopoly firm sets prices to maximize profits but the fringe firms limit the monopolist's market power.

Denote  $\sigma$  as the elasticity of substitution,  $\eta$  as the total industry constant price elasticity of demand, and let  $t_f$  be the tariff rate on foreign fringe firms. Let  $P$  be the CES price index for the industry,  $p_d$  be the price the monopolist sets, and  $p_f$  be the price of the fringe firm products. Let  $b$  be the preference asymmetry parameter for the fringe firms,  $k$  be the total expenditures on the products in the industry,  $q_d$  be the domestic monopolist quantity and  $q_f$  be the foreign firms quantity. Equations (10) through (12) describe the price index and demand functions.

$$P = \left( (p_d)^{1-\sigma} + b (p_f(1+t_f))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (10)$$

$$q_d = k (P)^\eta \left( \frac{p_d}{P} \right)^{-\sigma} \quad (11)$$

$$q_f = k (P)^\eta \left( \frac{p_f(1+t_f)}{P} \right)^{-\sigma} b \quad (12)$$

Profits of the monopolist are:

$$\pi_d = (p_d - c_d) q_d - f_d \quad (13)$$

The monopolist maximizes profits by choosing a price, taking the price of the fringe firms as given. Then the first order condition representing the

profit-maximizing choice of  $p_d$  is:

$$p_d = (p_d - c_d)(\sigma - (\sigma + \eta) \left(\frac{p_d}{P}\right)^{1-\sigma}) \quad (14)$$

## References

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