

STRUCTURAL EQUATIONS FOR PE MODELS  
IN GROUP 3  
(FOREIGN DIRECT INVESTMENT)

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**Abstract**

This paper presents the structural equations for the third group of industry-specific simulation models of changes in trade policy that are available for download on the USITC's PE Modeling Portal at [https://www.usitc.gov/data/pe\\_modeling/index.htm](https://www.usitc.gov/data/pe_modeling/index.htm).

The models described in this paper are the result of ongoing professional research of USITC staff and are solely meant to represent the professional research of individual authors. These papers are not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners. Please address correspondence to [david.riker@usitc.gov](mailto:david.riker@usitc.gov).

# 1 Introduction

These spreadsheet models address various aspects of foreign direct investment.

## 2 Model with Foreign Affiliate Sales

The first model is based on Helpman, Melitz and Yeaple (2004) as modified in Khachaturian and Riker (2016).  $Z_{P0}$  and  $Z_{X0}$  are calibrated to the initial equilibrium in the market.

$$Z_{P0} = \left(\frac{n_f}{n_d}\right) \left(\frac{f_P}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1} = \left(\frac{A_0}{D_0}\right) (1 - C_0^{1-\sigma})^{-\frac{\gamma}{\sigma-1}+1} \quad (1)$$

$$Z_{X0} = \left(\frac{n_f}{n_d}\right) \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1} = \left(\frac{M_0}{D_0}\right) C_0^\gamma + C_0^{1-\sigma+\gamma} (1 - C_0^{1-\sigma})^{\frac{\gamma}{\sigma-1}-1} Z_{P0} \quad (2)$$

The variables  $A_0$ ,  $M_0$ , and  $D_0$  represent the initial values of foreign affiliate sales, cross-border imports, and domestic sales.  $C_0$  is the initial relative variable cost of delivering foreign services supplied to the domestic market, including variable international trade costs.  $f_P$  is the incremental fixed cost of foreign affiliate supply,  $f_X$  is the fixed cost of cross-border trade, and  $f_D$  is the fixed cost of provision by domestic suppliers.  $n_d$  and  $n_f$  are the number of domestic and foreign firms that can potentially supply the domestic market.<sup>1</sup> Again,  $\sigma$  is the elasticity of substitution, and  $\gamma$  is the shape parameter of the Pareto distribution of firm-specific productivity levels.

Changes in the fixed costs of trade affect  $Z_P$  and  $Z_X$ . Equations (3) and (4) are the updating equations.

$$\frac{Z_P - Z_{P0}}{Z_{P0}} = \left(1 + \left(\frac{-\gamma}{\sigma-1} + 1\right) \left(\left(\frac{f_P - f_{P0}}{f_{P0}}\right) - \left(\frac{f_D - f_{D0}}{f_{D0}}\right)\right)\right) \quad (3)$$

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<sup>1</sup> $n_d$  and  $n_f$  are treated as exogenous variables in the partial equilibrium model.

$$\frac{Z_X - Z_{X0}}{Z_{X0}} = \left( 1 + \left( \frac{-\gamma}{\sigma - 1} + 1 \right) \left( \left( \frac{f_X - f_{X0}}{f_{X0}} \right) - \left( \frac{f_D - f_{D0}}{f_{D0}} \right) \right) \right) \quad (4)$$

The equilibrium values of foreign affiliate sales ( $A$ ), cross-border imports ( $M$ ), and domestic sales ( $D$ ) are defined by (5), (6), and (7).

$$A = \frac{E Z_P (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}-1}}{Z_P (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}} + Z_X C^{-\gamma} + 1} \quad (5)$$

$$M = \frac{E C^{1-\sigma} \left( Z_X C^{-\gamma+\sigma-1} - Z_P (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}-1} \right)}{Z_P (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}} + Z_X C^{-\gamma} + 1} \quad (6)$$

$$D = \frac{E}{Z_P (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}} + Z_X C^{-\gamma} + 1} \quad (7)$$

These three values sum to total expenditure on the industry in the market,  $E$ , which is held constant in model simulations.

### 3 Tariff-Jumping FDI Model

The second model is derived in Riker and Schreiber (2019), which is available for download on the PE Modeling Portal.

### 4 Export Platform FDI Model

The third model is derived in Riker and Schreiber (2019), which is available for download on the PE Modeling Portal.

## 5 Cross Border Ownership FDI Model

The fourth model is derived in Montgomery and Riker (2020), which is available for download on the PE Modeling Portal.

## 6 Model with Offshoring

In the fifth model, a partial equilibrium version of Grossman and Rossi-Hansberg (2008), there are two countries ( $d$  and  $f$ ) and two types of workers (low-skilled workers  $L$  and high-skilled workers  $H$ ). There is a continuum of tasks indexed by  $j$ . The model assumes that the cost of offshoring the two types of tasks,  $1 + j$ , are both uniformly distributed between zero and one.<sup>2</sup> The model calculates changes in the share of tasks offshored by skill level ( $J_H$  and  $J_L$ ) and domestic employment by skill level ( $E_H$  and  $E_L$ ), as well as the change in the product price ( $p$ ). The model quantify the economic effects of exogenous changes in wage rates in the two countries ( $w_d$  and  $w_f$ ) and the relative productivity of foreign workers within each skill type ( $\lambda_L$  and  $\lambda_H$ ). The relative productivity of low-skilled workers overall ( $\gamma$ ) is calibrated within the model.

$$J_L = \frac{w_d}{w_f \lambda_L} - 1 \quad (8)$$

$$J_H = \frac{w_d}{w_f \lambda_H} - 1 \quad (9)$$

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<sup>2</sup>Grossman and Rossi-Hansberg (2008) do not assume a specific form for this distribution, but it is necessary to specify a distribution to generate a quantitative estimates in the model.

$$p^{1-\theta} = \left( w_d (1 - J_H) + \lambda_H w_f \left( \frac{1}{2} J_H^2 + J_H \right) \right)^{1-\theta} + \gamma \left( w_d (1 - J_L) + \lambda_L w_f \left( \frac{1}{2} J_L^2 + J_L \right) \right)^{1-\theta} \quad (10)$$

$$E_L = a_L k p^\eta \left( \frac{\gamma \left( w_d (1 - J_L) + \lambda_L w_f \left( \frac{1}{2} J_L^2 + J_L \right)^{1-\theta} \right)}{p^{1-\theta}} \right)^{-\theta} J_L \quad (11)$$

$$E_H = a_H k p^\eta \left( \frac{\left( w_d (1 - J_H) + \lambda_H w_f \left( \frac{1}{2} J_H^2 + J_H \right)^{1-\theta} \right)}{p^{1-\theta}} \right)^{-\theta} J_H \quad (12)$$

## References

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