STRUCTURAL EQUATIONS FOR PE MODELS
IN GROUP 2 (IMPERFECT COMPETITION)

David Riker and Samantha Schreiber
U.S. International Trade Commission, Office of Economics

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Abstract
This note presents the structural equations for the second group of PE simulation models of changes in trade policy. Euler Method spreadsheet versions of these models are available at https://www.usitc.gov/data/pe_modeling/index.htm.

The models described in this documentation are the result of ongoing professional research of USITC staff and are solely meant to represent the professional research of individual authors. These papers are not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners. Please address correspondence to david.riker@usitc.gov.
1 Introduction

There are several PE models with imperfect competition that have been translated into a user-friendly spreadsheet format.

2 Bertrand Differentiated Products Tariff Model

The first variant of the model relaxes the perfect competition assumption by allowing firms to choose their price to maximize profits. This model is applicable to highly concentrated industries in which imports are an important source of competition and profits are at stake. The Bertrand model assumes non-nested CES demand functions with elasticity of substitution $\sigma$. Let $d$ be the index for the domestic firm, $s$ be the index for the foreign firm subject to tariffs, and $n$ be the index for the non-subject foreign firm. $P$ is the CES price index for the industry, $p_j$ are the prices of the products of firm $j \in \{d, s, n\}$, $b_j$ is a preference asymmetry parameter, $t_s$ is the tariff applied to the subject imports $q_s$, $c_j$ is the constant marginal cost, $f_j$ is the fixed cost, and $\eta$ is the price elasticity of total industry demand. Equations (1) through (5) describe the price index and demand functions.

$$P = \left((p_d)^{1-\sigma} + b_s(p_s(1 + t_s))^{1-\sigma} + b_n(p_n)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

See Riker (2018) for more information on the Bertrand model and simulated results.
Profits of firm \( j \in \{d, s, n\} \) are:

\[
\pi_j(p_j) = (p_j - c_j) q_j - f_j
\]

(5)

In Bertrand-style competition, each firm maximizes profits by choosing a price, taking the prices of the other firms as given. Then the first order condition representing the profit-maximizing choice of \( p_j \) is:

\[
p_j = (p_j - c_j) \left( \sigma - (\sigma + \eta) \frac{b_j(p_j(1 + t_j))^{1-\sigma}}{\sum_k b_k(p_k(1 + t_k))^{1-\sigma}} \right)
\]

(6)

3 Cournot Tariff Model

The second variant is a Cournot Competition model. This model is also applicable to highly concentrated industries in which imports are an important source of competition but unlike the Bertrand model, this model assumes that foreign and domestic products are perfect substitutes. Suppose there

\[
q_d = k \ (P)^\eta \ (\frac{p_d}{P})^{-\sigma}
\]

(2)

\[
q_s = k \ b_s \ (P)^\eta \ (\frac{p_s(1 + t_s)}{P})^{-\sigma}
\]

(3)

\[
q_n = k \ b_n \ (P)^\eta \ (\frac{p_n}{P})^{-\sigma}
\]

(4)
are three firms, a domestic firm \((d)\), a foreign firm subject to tariffs \((s)\), and a foreign firm not subject to tariffs \((n)\). Denote \(P\) as the market price, \(\eta\) as the constant price elasticity of demand, and \(\beta\) as a constant calibrated to the size of the market. Then for \(k \in \{d, s, n\}\), consumer demand takes on the following functional form:

\[
\sum_k q_k = \beta \left( P \right)^\eta
\]  

(7)

Profits of firm \(j \in \{d, s, n\}\) are:

\[
\pi_j = (P - c_j (1 + t_j)) q_j - f_j
\]  

(8)

In Cournot-style competition, each firm maximizes profits by choosing a quantity, taking the output of the other firms as given. Then the first order condition representing the profit-maximizing choice of \(q_j\) is:

\[
\left( \frac{1}{\beta} \left( \sum_k q_k \right)^{\frac{1}{\eta}} - c_j (1 + t_j) \right) + \left( \frac{1}{\beta} \left( \sum_k q_k \right)^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q_j}{\sum_k q_k} \right) \right) = 0
\]  

(9)

4 Monopoly Tariff Model

The third variant is a domestic monopoly model with a foreign competitive fringe. This model is applicable to industries with a dominant firm who has a large market share and several smaller price-taking firms called fringe firms.
The dominant monopoly firm sets prices to maximize profits but the fringe firms limit the monopolist’s market power.

Denote \( \sigma \) as the elasticity of substitution, \( \eta \) as the total industry constant price elasticity of demand, and let \( t_f \) be the tariff rate on foreign fringe firms. Let \( P \) be the CES price index for the industry, \( p_d \) be the price the monopolist sets, and \( p_f \) be the price of the fringe firm products. Let \( b \) be the preference asymmetry parameter for the fringe firms, \( k \) be the total expenditures on the products in the industry, \( q_d \) be the domestic monopolist quantity and \( q_f \) be the foreign firms quantity. Equations (10) through (12) describe the price index and demand functions.

\[
P = \left( (p_d)^{1-\sigma} + b (p_f (1 + t_f))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{10}
\]

\[
q_d = k (P)\eta \left( \frac{p_d}{P} \right)^{-\sigma} \tag{11}
\]

\[
q_f = k (P)\eta \left( \frac{p_f (1 + t_f)}{P} \right)^{-\sigma} b \tag{12}
\]

Profits of the monopolist are:

\[
\pi_d = (p_d - c_d) q_d - f_d \tag{13}
\]

The monopolist maximizes profits by choosing a price, taking the price of the fringe firms as given. Then the first order condition representing the
profit-maximizing choice of \( p_d \) is:

\[
p_d = (p_d - c_d)(\sigma - (\sigma + \eta) \left( \frac{p_d}{P} \right)^{1-\sigma})
\]  

(14)

5 Fixed Costs and Firm Heterogeneity

The fourth variant is a two-country model of international trade with firm heterogeneity. Within the industry, there is a continuum of firms supplying differentiated products, with constant elasticity of substitution \( \sigma \). The firms vary in their unit labor requirements. Firm-specific productivity has a Pareto distribution with shape parameter \( \gamma \). There is a fixed cost of production \( f_D \) and an incremental fixed cost of exporting \( f_X \). \( D \) is the aggregate value of domestic shipments in the market, integrated over the mass \( n_D \) of domestic suppliers.

\[
D = k \left( \frac{n_d}{n_d + n_f (\tau)^{-\gamma} \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{2}} + 1} \right)
\]  

(15)

\( M \) is the aggregate value of imports into the market, integrated over the mass \( n_F \) of foreign suppliers. \( k \) is total industry expenditures in the market, and \( \tau \) is a variable trade cost on imports.

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\(^2\) The model adopts the standard assumption in the literature that \( \gamma > \sigma - 1 \).

\(^3\) The model assumes that the number of firms that participate in the market is endogenously determined but the numbers of potential market participants, \( n_d \) and \( n_f \), are exogenous.
\[ M = k \left( \frac{n_f (\tau)^{-\gamma} \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{\sigma}+1}}{n_d + n_f (\tau)^{-\gamma} \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{\sigma}+1}} \right) \]  

Equation (17) is the ratio of the value of imports to the value of domestic shipments.

\[ \frac{M}{D} = (\tau)^{-\gamma} \left( \frac{n_f}{n_d} \right) \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{\sigma}+1} \]  

Next, we define \( Z_0 = \left( \frac{n_f}{n_d} \right) \left( \frac{f_X}{f_D} \right)^{\frac{\gamma}{\sigma}+1} \). The model calibrates \( Z_0 \) based on the ratio of the value of imports to the value of domestic shipments in the initial equilibrium and initial trade costs.

\[ Z_0 = \left( \frac{M_0}{D_0} \right) \left( \frac{\tau_0}{\sigma} \right)^{\gamma} \]  

\( \tau_0 \) is the initial trade cost factor, \( M_0 \) is the initial value of imports, and \( D_0 \) is the initial value of domestic shipments. An increase in the fixed cost of exporting \( f_X \) decreases \( Z \) and relative expenditure on imports, and an increase in the fixed cost of domestic production \( f_D \) increases \( Z \) and relative expenditure on imports.

\[ Z = Z_0 \left( 1 + \left( \frac{-\gamma}{\sigma} + 1 \right) \left( \left( \frac{f_X - f_{X0}}{f_{X0}} \right) - \left( \frac{f_D - f_{D0}}{f_{D0}} \right) \right) \right) \]  

The model simulates the effects of changes in the fixed costs (\( f_X \) and \( f_D \)) and the variable trade cost (\( \tau \)) on the value of imports (\( M \)) and the value of
domestic shipments \( D \), based on (X) and (X).

\[
D = D_0 \left( \frac{1 + Z_0 (\tau_0)^{-\gamma}}{1 + Z (\tau)^{-\gamma}} \right) \tag{20}
\]

\[
M = M_0 \left( \frac{1 + Z_0 (\tau_0)^{-\gamma}}{1 + Z (\tau)^{-\gamma}} \right) \left( \frac{Z (\tau)^{-\gamma}}{Z_0 (\tau_0)^{-\gamma}} \right) \tag{21}
\]

References