STRUCTURAL EQUATIONS FOR PE MODELS IN GROUP 2 (IMPERFECT COMPETITION)

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Abstract

This note presents the structural equations for the second group of PE simulation models of changes in trade policy. Euler Method spreadsheet versions of these models are available at `https://www.usitc.gov/data/pe_modeling/index.htm`.

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1 Introduction

There are several PE models with imperfect competition that have been translated into a user-friendly spreadsheet format.

2 Bertrand Differentiated Products Tariff Model

The first variant of the model relaxes the perfect competition assumption by allowing firms to choose their price to maximize profits. This model is applicable to highly concentrated industries in which imports are an important source of competition and profits are at stake.[1] The Bertrand model assumes non-nested CES demand functions with elasticity of substitution σ. Let d be the index for the domestic firm, s be the index for the foreign firm subject to tariffs, and n be the index for the non-subject foreign firm. P is the CES price index for the industry, p_j are the prices of the products of firm $j \in \{d, s, n\}$, b_j is a preference asymmetry parameter, t_s is the tariff applied to the subject imports q_s, c_j is the constant marginal cost, f_j is the fixed cost, and η is the price elasticity of total industry demand. Equations (1) through (5) describe the price index and demand functions.

$$P = \left((p_d)^{1-\sigma} + b_s(p_s(1+t_s))^{1-\sigma} + b_n(p_n)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{1}$$

[1]See Riker (2018) for more information on the Bertrand model and simulated results.

$$q_d = k \ (P)^\eta \ \left(\frac{p_d}{P}\right)^{-\sigma} \tag{2}$$

$$q_s = k \ b_s \ (P)^\eta \ \left(\frac{p_s(1+t_s)}{P}\right)^{-\sigma} \tag{3}$$

$$q_n = k \ b_n \ (P)^\eta \ \left(\frac{p_n}{P}\right)^{-\sigma} \tag{4}$$

Profits of firm $j \in \{d, s, n\}$ are:

$$\pi_j(p_j) = (p_j - c_j) \ q_j - f_j \tag{5}$$

In Bertrand-style competition, each firm maximizes profits by choosing a price, taking the prices of the other firms as given. Then the first order condition representing the profit-maximizing choice of p_j is:

$$p_j = (p_j - c_j)\Big(\sigma - (\sigma+\eta)\frac{b_j(p_j(1+t_j))^{1-\sigma}}{\sum_k b_k(p_k(1+t_k))^{1-\sigma}}\Big) \tag{6}$$

3 Cournot Tariff Model

The second variant is a Cournot Competition model. This model is also applicable to highly concentrated industries in which imports are an important source of competition but unlike the Bertrand model, this model assumes that foreign and domestic products are perfect substitutes. Suppose there are three firms, a domestic firm (d), a foreign firm subject to tariffs (s), and a foreign firm not subject to tariffs (n). Denote P as the market price, η as the constant price elasticity of demand, and β as a constant calibrated to the size of the market. Then for $k \in \{d, s, n\}$, consumer demand takes on the following functional form:

$$\sum_k q_k = \beta \ (P)^{\eta} \tag{7}$$

Profits of firm $j \in \{d, s, n\}$ are:

$$\pi_j = (P - c_j(1 + t_j)) \ q_j - f_j \tag{8}$$

In Cournot-style competition, each firm maximizes profits by choosing a quantity, taking the output of the other firms as given. Then the first order condition representing the profit-maximizing choice of q_j is:

$$\Big(\big(\frac{1}{\beta}\big(\sum_k q_k\big)\big)^{\frac{1}{\eta}} - c_j \ (1 + t_j)\Big) + \big(\frac{1}{\beta}\big(\sum_k q_k\big)\big)^{\frac{1}{\eta}} \ \frac{1}{\eta}\big(\frac{q_j}{\sum_k q_k}\big) = 0 \tag{9}$$

4 Monopoly Tariff Model

The third variant is a domestic monopoly model with a foreign competitive fringe. This model is applicable to industries with a dominant firm who has a large market share and several smaller price-taking firms called fringe firms.

The dominant monopoly firm sets prices to maximize profits but the fringe firms limit the monopolist's market power.

Denote σ as the elasticity of substitution, η as the total industry constant price elasticity of demand, and let t_f be the tariff rate on foreign fringe firms. Let P be the CES price index for the industry, p_d be the price the monopolist sets, and p_f be the price of the fringe firm products. Let b be the preference asymmetry parameter for the fringe firms, k be the total expenditures on the products in the industry, q_d be the domestic monopolist quantity and q_f be the foreign firms quantity. Equations (10) through (12) describe the price index and demand functions.

$$P = \left((p_d)^{1-\sigma} + b\ (p_f(1+t_f))^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{10}$$

$$q_d = k\ (P)^{\eta}\ \left(\frac{p_d}{P}\right)^{-\sigma} \tag{11}$$

$$q_f = k\ (P)^{\eta}\ \left(\frac{p_f(1+t_f)}{P}\right)^{-\sigma}\ b \tag{12}$$

Profits of the monopolist are:

$$\pi_d = (p_d - c_d)\ q_d - f_d \tag{13}$$

The monopolist maximizes profits by choosing a price, taking the price of the fringe firms as given. Then the first order condition representing the profit-maximizing choice of p_d is:

$$p_d = (p_d - c_d)(\sigma - (\sigma + \eta) \left(\frac{p_d}{P}\right)^{1-\sigma}) \tag{14}$$

5 Fixed Costs and Firm Heterogeneity

The fourth variant is a two-country model of international trade with firm heterogeneity. Within the industry, there is a continuum of firms supplying differentiated products, with constant elasticity of substitution σ. The firms vary in their unit labor requirements. Firm-specific productivity has a Pareto distribution with shape parameter γ.[2] There is a fixed cost of production f_D and an incremental fixed cost of exporting f_X. D is the aggregate value of domestic shipments in the market, integrated over the mass n_D of domestic suppliers.[3]

$$D = k \left(\frac{n_d}{n_d + n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1}} \right) \tag{15}$$

M is the aggregate value of imports into the market, integrated over the mass n_F of foreign suppliers. k is total industry expenditures in the market, and τ is a variable trade cost on imports.

[2] The model adopts the standard assumption in the literature that $\gamma > \sigma - 1$.

[3] The model assumes that the number of firms that participate in the market is endogenously determined but the numbers of potential market participants, n_d and n_f, are exogenous.

$$M = k \left(\frac{n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1}}{n_d + n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1}} \right) \tag{16}$$

Equation (17) is the ratio of the value of imports to the value of domestic shipments.

$$\frac{M}{D} = (\tau)^{-\gamma} \left(\frac{n_f}{n_d}\right) \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1} \tag{17}$$

Next, we define $Z_0 = \left(\frac{n_f}{n_d}\right) \left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1}$. The model calibrates Z_0 based on the ratio of the value of imports to the value of domestic shipments in the initial equilibrium and initial trade costs.

$$Z_0 = \left(\frac{M_0}{D_0}\right) \ (\tau_0)^{\gamma} \tag{18}$$

τ_0 is the initial trade cost factor, M_0 is the initial value of imports, and D_0 is the initial value of domestic shipments. An increase in the fixed cost of exporting f_X decreases Z and relative expenditure on imports, and an increase in the fixed cost of domestic production f_D increases Z and relative expenditure on imports.

$$Z = Z_0 \ \left(1 + \left(\frac{-\gamma}{\sigma-1} + 1\right) \left(\left(\frac{f_X - f_{X0}}{f_{X0}}\right) - \left(\frac{f_D - f_{D0}}{f_{D0}}\right)\right)\right) \tag{19}$$

The model simulates the effects of changes in the fixed costs (f_X and f_D) and the variable trade cost (τ) on the value of imports (M) and the value of domestic shipments (D), based on (X) and (X).

$$D = D_0 \ \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}} \right) \tag{20}$$

$$M = M_0 \ \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}} \right) \ \left(\frac{Z(\tau)^{-\gamma}}{Z_0(\tau_0)^{-\gamma}} \right) \tag{21}$$

References

D. Riker. Modeling the financial impact of tariffs in concentrated product markets. *U.S. International Trade Commission Economics Working Paper*, No. 2018-10-B, 2018.